Central Bank Learning, Terms of Trade Shocks & Currency Risk: Should Only Inflation Matter for Monetary Policy?*

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Abstract

This paper examines the role of interest rate policy in a small open economy subject to terms of trade shocks, and time-varying currency risks. The private sector makes optimal decisions in an intertemporal non-linear setting with rational, forward-looking expectations. In contrast, the monetary authority chooses an optimal interest rate reaction function, given a loss function that is conditional on the state of the economy and given its "least-squares learning" about the evolution of inflation and exchange rate depreciation. The simulation results of the effects of different policy scenarios on welfare show that, on balance, the prefered stance should be strict inflation targeting.

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Key words: policy targets, central bank learning, parameterized expectations

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1 Introduction

This paper examines the role of interest rate policy in a small open economy subject to terms of trade shocks, and time-varying currency risks. A central bank committed to low inflation controls neither the terms of trade nor the evolution of currency risk, both of which condition the response of inflation to its policy instruments. In this context, the best the central bank can do is to "learn" the effects indirectly, by frequently "updating" estimates of inflation dynamics and "re-adjusting" its policy rules accordingly.

However, when an economy is subjected to large adverse external shocks and the exchange rate depreciates rapidly, it should not be surprising if a central bank also comes under strong pressure to incorporate exchange rate volatility targets in its policy objectives. Should exchange rate changes then be included as one of the monetary policy targets along with inflation targets?

Much of the discussion of monetary policy is framed by the well-known Taylor (1993, 1999) rule, whereby interest rates respond to their own lag, as well as to deviations of inflation and output from respective targets. Taylor (1993) points out that this "rule" need not be a mechanical formula, but something which can be operated "informally", with recognition of the "general instrument responses which underlie the policy rule". Not surprisingly, the specification of this rule, which reflect the underlying objectives of monetary policy, has been the subject of considerable controversy.[1]

In a closed-economy setting, Christiano and Gust (1999), for example, argue that only the inflation variable should appear as a target. Rotemberg and Woodford (1998) concur, but they argue that a higher average rate of inflation is required for monetary policy to be effective over the medium to long term. They base their argument on the zero lower bound for the nominal interest rate, since at very low inflation rates there is little room for this instrument to manoeuvre.[2]

In an open economy setting, McCallum (2000) takes issue with the Rotemberg and Woodford "policy ineffectiveness" argument under low inflation and zero "lower bounds" for nominal interest rates. McCallum argues that the central bank always has at its disposal a second tool, the exchange rate, so if the economy is stuck at a very low interest rate, there is the option of currency intervention. Christiano (2000) disagrees: McCallum's argument rests on the assumption that currency depreciation is effective. Furthermore, the

[1] Recent technical papers on all aspects of the Taylor rule may be found on the web page, http://www.stanford.edu/~johntayl/PolRulLink.htm#Technical%20articles

[2] Erceg, Henderson and Levin (2000) argued that output deviations should also appear in the Taylor rule, but the output measure should be deviations of actual output from the level of output generated by a flexible-price economy.

central bank must be willing to undermine public "confidence" that it stands ready to cut interest rates in the event of major adverse shocks.

For small emerging market economies, Taylor (2000) contends that policy rules that focus on a "smoothed inflation measure and real output" and which do not "try to react too much" to the exchange rate might work well. However, he leaves open the question of a role for the exchange rate. Ball (1999) argues that inflation targeting "can be dangerous" in an open economy setting because exchange rate changes have a direct effect on inflation via changes in import prices. Hence, adoption of a strict inflation targeting stance can result in large output variations. More recently, Gali and Monacelli (2002) found, in a small open economy setup with sticky price setting behavior, that domestic inflation targeting dominates, from a welfare point of view, both CPI inflation targeting and an exchange-rate peg. They base their argument on the "excess smoothness" induced in the exchange rate by CPI targeting or an exchange rate peg. This smoothness, in combination with the assumed stickiness in nominal prices, prevents relative prices from adjusting "sufficiently fast", thus causing "a significant deviation from the first best allocation" [Gali and Monacelli (2002), p.2].

However, practically all of these studies are based on *linear* stochastic and dynamic general equilibrium representations, or *linearized* approximations of nonlinear models. The Taylor-type feedback rules are either imposed or derived by linear quadratic optimization. While these approaches may be valid if the shocks impinging on the economy are indeed "small" and "symmetric" deviations from a steady state, they may be inappropriate if the shocks are large, persistent, and asymmetric, as they are in many highly open economies.

Furthermore, few if any of these studies incorporate "learning" on the part of the monetary authority itself. Bullard and Mitra (2002) incorporate private sector "learning" of the specific Taylor rules used by the central bank in the Rotemberg-Woodford closed economy framework. They argue for Taylor rules based on *expectations* of *current* inflation from target levels, rather than rules based on lagged values or forecasts further into the future.

In contrast to Bullard and Mitra (2001), we assume that the private sector uses the true, stochastic dynamic, nonlinear model for formulating its own "laws of motion" for consumption, investment, and trade, with forward-looking rational expectations. In this analysis, the monetary policy authority learns the "laws of motion" of inflation dynamics from past data, through continuously-updated least squares regression. From the results of these regressions, the monetary authority obtains an optimal interest rate feedback rule based on linear quadratic optimization, using weights in the objective function for inflation which can vary with current conditions. The monetary authority is thus "boundedly rational", in the sense of Sargent (1999), with "rational" describing the use of least squares, and "bounded" meaning model misspecification.

Our results show that when a central banker is all-knowing and acts to optimize the intertemporal welfare of the consumer, there is not much difference in terms of welfare outcomes between fixed and flexible inflation targeting. In contrast, if the central bank decides to incorporate, in addition to inflation, exchange rate dynamics in its learning and policy objectives, and prices are sticky, it does so at some welfare costs. In a learning environment, there is always the risk that the "perceived" laws of motion lag behind the actual laws of motion. Hence expanding the range of policy objectives may increase overall volatility and reduce welfare. For this reason, strict inflation targeting dominates monetary policy based on multiple targets in an environment with central bank learning and sticky prices.

The next section describes the theoretical structure of the model for the private sector and the nature of the monetary authority "learning". The third section discusses the calibration as well as the solution method, while the fourth section analyzes the simulation results of the model. The last section concludes.

2 The Model

The framework of analysis contains two modules - a module which describes the behavior of the private sector and a module which describes the behavior of the central bank.

2.1 Private Sector Behavior

The private sector is assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models.

2.1.1 Consumption

The utility function for the private sector "representative agent" is given by the following function:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{1}$$

where C is the aggregate consumption index and γ is the coefficient of relative risk aversion. Unless otherwise specified, upper case variables denote the levels of the variables while lower case letters denote logarithms of the same variables. The exception is the nominal interest rate denoted as i.

The representative agent as "household/firm" optimizes the following intertemporal welfare function, with an endogenous discount factor:

$$
\begin{aligned}
W_t &= \mathbf{E}\left[\sum_{i=0}^{\infty} \vartheta_{t+i} U(C_{t+i})\right] && (2)\\
\vartheta_{t+1+i} &= [1+\overline{C}_t]^{-\beta} \cdot \vartheta_{t+i} && (3)\\
\vartheta_t &= 1 && (4)
\end{aligned}
$$

where $\mathbf{E}_t$ is the expectations operator, conditional on information available at time t, while β approximates the elasticity of the endogenous discount factor ϑ with respect to the average consumption index, $\overline{C}$. Endogenous discounting is due to Uzawa (1968) and Mendoza (2000) states that endogenous discounting is needed for the model to produce well-behaved dynamics with deterministic stationary equilibria.[3]

The specification used in this paper is due to Schmitt-Grohé and Uribe (2001). In our model, an individual agent's discount factor does not depend on their own consumption, but rather their discount factor depends on the average level of consumption. Schmitt-Grohé and Uribe (2001) argue that this simplification reduces the equilibrium conditions by one Euler equation and one state variable, over the standard model with endogenous discounting, it greatly facilitates the computation of the equilibrium dynamics, while delivering "virtually identical" predictions of key macroeconomic variables as the standard endogenous-discounting model.[4] In equilibrium, of course, the individual consumption index and the average consumption index are identical. Hence,

$$C_t = \overline{C}_t \tag{5}$$

The consumption index is a composite index of non-tradeable goods n and tradeable goods f :

$$C_t = \left(C_t^f\right)^{\alpha_f} (C_t^n)^{1-\alpha_f} \tag{6}$$

where α_f is the proportion of traded goods. Given the aggregate consumption expenditure constraint,

$$P_t C_t = P_t^f C_t^f + P_t^n C_t^n \tag{7}$$

[3]Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

[4]Schmitt-Grohé and Uribe (2001) argue that if the reason for introducing endogenous discounting is solely for introducing stationarity, "computational convenience" should be the decisive factor for modifying the standard Uzawa-type model. Kim and Kose (2001) reached similar conclusions.

and the definition of the real exchange rate,

$$Z_t = \frac{P_t^f}{P_t^n} \tag{8}$$

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate Z:

$$C_t^f = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{-1+\alpha_f} Z_t^{-1+\alpha_f} C_t \tag{9}$$

$$C_t^n = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{\alpha_f} Z_t^{\alpha_f} C_t \tag{10}$$

Similarly, we can express the consumption of traded goods as a composite index of the consumption of export goods, C^x, and import goods C^m:

$$C_t^f = (C_t^x)_t^{\alpha_x} (C_t^m)^{1-\alpha_x} \tag{11}$$

where α_x is the proportion of export goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

$$P_t^f C_t^f = P_t^m C_t^m + P_t^x C_t^x \tag{12}$$

where P^x and P^m are the prices of export and import type goods respectively. Defining the terms of trade index J as:

$$J = \frac{P^x}{P^m} \tag{13}$$

yields the demand for export and import goods as functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$C_t^x = \left(\frac{1-\alpha_x}{\alpha_x}\right)^{-1+\alpha_x} J_t^{-1+\alpha_x} C_t^f \tag{14}$$

$$C_t^m = \left(\frac{1-\alpha_x}{\alpha_x}\right)^{\alpha_x} J_t^{\alpha_x} C_t^f \tag{15}$$

2.1.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$Y_t^x = A_t^x (K_{t-1}^x)^{1-\theta_x} \tag{16}$$

$$Y_t^m = A_t^m (K_{t-1}^m)^{1-\theta_m} \tag{17}$$

where A^x, A^m represents the labour factor productivity terms[5] in the production of export and import goods, and $(1-\theta_x)$, $(1-\theta_m)$ are the coefficients of the capital K^x and K^m respectively. The time subscripts $(t-1)$ indicates that they are the beginning-of-period values. The production of non-traded goods, which is usually in services, is given by the labour productivity term, A_t^n :

$$Y_t^n = A_t^n \tag{18}$$

Capital in each sector has the respective depreciation rates, δ_x and δ_m, and evolves according to the following identities:

$$K_t^x = (1-\delta_x)K_{t-1}^x + I_t^x \tag{19}$$
$$K_t^m = (1-\delta_m)K_{t-1}^m + I_t^m \tag{20}$$

where I_t^x and I_t^m represents investment in each sector.

2.1.3 Budget Constraint

The budget constraint faced by the household/firm representative agent is:

$$P_tC_t = \Pi_t + S_t\left[L_t^* - L_{t-1}^*(1+i_{t-1}^* + \theta_{t-1})\right] - \left[B_t - B_{t-1}(1+i_{t-1})\right] \tag{21}$$

where S is the exchange rate (defined as domestic currency per foreign), L_t^* is foreign debt in foreign currency, and B_t is domestic debt in domestic currency. Profits Π is defined by the following expression:

$$\begin{aligned}\Pi_t = \; & P_t^x\left[A_t^x\left(K_{t-1}^x\right)^{1-\theta_x} - \frac{\phi_x}{2K_{t-1}^x}\left(I_t^x\right)^2 - I_t^x\right] \\ & + P_t^m\left[A_t^m(K_{t-1}^m)^{1-\theta_m} - \frac{\phi_m}{2K_{t-1}^m}\left(I_t^m\right)^2 - I_t^m\right] + P_t^nA_t^n \end{aligned} \tag{22}$$

The aggregate resource constraint shows that the firm faces quadratic adjustment costs when they accumulate capital, with these costs given by the terms $\frac{\phi_x}{2K_{t-1}^x}\left(I_t^x\right)^2$ and $\frac{\phi_m}{2K_{t-1}^m}\left(I_t^m\right)^2$.

As shown, the household/firm lends to the domestic government and accumulate bonds B which pay the nominal interest rate i. They also borrow internationally and accumulate international debt L^* at the fixed rate i^*, including a time-varying risk premium $\theta_{t.}$. The evolution of the currency risk term θ_t is modelled as a GARCH process:

$$\theta_t = \xi_0 + \xi_1\theta_{t-1} + \xi_2\eta_{t-1}^2 \tag{23}$$

[5] Since the representative agent determines both consumption and production decisions, we have simplified the analysis by abstracting from issues about labour-leisure choice and wage determination.

We also assume, that the shocks, η_t are drawn from a t-distribution to better capture the empirical leptokurtic characteristics of risk.[6]

2.1.4 Euler Equations

The household/firm optimizes the expected value of the utility of consumption (2) subject to the budget constraint defined in (21) and (22) and the constraints in (19) and (20).

$$
\begin{aligned}
Max \quad : \quad Ł = \mathbf{E}_t \sum_{i=0}^{\infty} \vartheta_{t+i} \{ U(C_{t+i}) \\
& -\Lambda_{t+i}[C_{t+i} - \frac{P^x_{t+i}}{P_{t+i}} \left(A^x_{t+i}(K^x_{t-1+i})^{1-\theta_x} - \frac{\phi_x}{2K^x_{t-1+i}} \left(I^x_{t+i}\right)^2 - I^x_{t+i} \right) \\
& -\frac{P^m_{t+i}}{P_{t+i}} \left(A^m_{t+i}(K^m_{t-1+i})^{1-\theta_m} - \frac{\phi_m}{2K^m_{t-1+i}} \left(I^m_{t+i}\right)^2 - I^m_{t+i} \right) - \frac{P^n_{t+i}}{P_{t+i}} A^n_{t+i} \\
& -\frac{S_{t+i}}{P_{t+i}} \left(L^*_{t+i} - L^*_{t-1+i}(1 + i^*_{t-1+i} + \theta_{t-1+i}) \right) + \frac{1}{P_{t+i}} \left(B_{t+i} - B_{t-1+i}(1 + i_{t-1+i}) \right)] \\
& -Q^x_{t+i} \left[K^x_{t+i} - I^x_{t+i} - (1 - \delta_x) K^x_{t-1+i} \right] \\
& -Q^m_{t+i} \left[K^m_{t+i} - I^m_{t+i} - (1 - \delta_m) K^m_{t-1+i} \right] \}
\end{aligned}
$$

The variable Λ is the familiar Lagrangean multiplier representing the marginal utility of wealth. The terms Q^x and Q^m, known as Tobin's Q, represent the Lagrange multipliers for the evolution of capital in each sector - they are the "shadow prices" for new capital. Maximizing the Lagrangean with respect to $C_t, L^*_t, B_t, K^x_t, K^m_t, I^x_t, I^m_t$ yields the following first order conditions:

$$
\begin{aligned}
U'(C_t) - \Lambda_t &= 0 \\
\vartheta_t \Lambda_t \frac{S_t}{P_t} - \mathbf{E}_t \left[\vartheta_{t+1} \Lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + i^*_t + \theta_t) \right] &= 0 \\
-\vartheta_t \Lambda_t \frac{1}{P_t} + \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{1}{P_{t+1}} (1 + i_t) &= 0
\end{aligned}
$$

[6] The focus here is clearly on the design of policy in respond to exogenously determined shocks on the exchange rate. The analysis can be further complicated by allowing currency risk to be endogenously conditioned by changes in domestic internal and foreign external debt as well as inflation and exchange rate changes.

$$\begin{bmatrix} -\vartheta_t Q_t^x \\ +\mathbf{E}_t \vartheta_{t+1} Q_{t+1}^x (1-\delta_x) \end{bmatrix} + \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^x}{P_{t+1}} \begin{bmatrix} A_{t+1}^x (1-\theta_x)(K_t^x)^{-\theta_x} \\ +\frac{\phi_x \left(I_{t+1}^x\right)^2}{2(K_t^x)^2} \end{bmatrix} = 0$$

$$\begin{bmatrix} -\vartheta_t Q_t^m \\ +\mathbf{E}_t \vartheta_{t+1} Q_{t+1}^m (1-\delta_m) \end{bmatrix} + \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^m}{P_{t+1}} \begin{bmatrix} A_{t+1}^m (1-\theta_m)(K_t^m)^{-\theta_m} \\ +\frac{\phi_m \left(I_{t+1}^m\right)^2}{2(K_t^m)^2} \end{bmatrix} = 0$$

$$-\vartheta_t \Lambda_t \frac{P_t^x}{P_t} \left(\frac{\phi_x I_t^x}{K_{t-1}^x} + 1 \right) + \vartheta_t Q_t^x = 0$$

$$-\vartheta_t \Lambda_t \frac{P_t^m}{P_t} \left(\frac{\phi_m I_t^m}{K_{t-1}^m} + 1 \right) + \vartheta_t Q_t^m = 0$$

These equations can then be re-expressed as:

$$\Lambda_t = U'(C_t) \tag{24}$$

$$\vartheta_t U'(C_t) = \mathbf{E}_t \vartheta_{t+1} U'(C_{t+1})(1 + i_t - \pi_{t+1}) \tag{25}$$

$$\mathbf{E}_t(s_{t+1} - s_t) = i_t - i_t^* - \theta_t \tag{26}$$

$$\begin{bmatrix} \vartheta_t Q_t^x \\ -\mathbf{E}_t \vartheta_{t+1} Q_{t+1}^x (1-\delta_x) \end{bmatrix} = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^x}{P_{t+1}} \begin{bmatrix} A_{t+1}^x (1-\theta_x)(K_t^x)^{-\theta_x} \\ +\frac{\phi_x \left(I_{t+1}^x\right)^2}{2(K_t^x)^2} \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} \vartheta_t Q_t^m \\ -\mathbf{E}_t \vartheta_{t+1} Q_{t+1}^m (1-\delta_m) \end{bmatrix} = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^m}{P_{t+1}} \begin{bmatrix} A_{t+1}^m (1-\theta_m)(K_t^m)^{-\theta_m} \\ +\frac{\phi_m \left(I_{t+1}^m\right)^2}{2(K_t^m)^2} \end{bmatrix} \tag{28}$$

$$I_t^x = \frac{1}{\phi_x} \left(\frac{P_t}{P_t^x} \frac{Q_t^x}{\Lambda_t} - 1 \right) K_{t-1}^x \tag{29}$$

$$I_t^m = \frac{1}{\phi_m} \left(\frac{P_t}{P_t^m} \frac{Q_t^m}{\Lambda_t} - 1 \right) K_{t-1}^m \tag{30}$$

where $\Delta p_{t+1} = \log(P_{t+1}/P_t)$ is the per period inflation, s is the logarithm of the nominal exchange rate S and $(\mathbf{E}_t s_{t+1} - s_t)$ is the expected rate of exchange rate depreciation.

Equation (25) is the typical Euler equation for consumption. Using the utility function in (1) yields the consumption function:

$$C_t = \mathbf{E}_t \left[(1 + i_t - \pi_{t+1}) \vartheta_{t+1} C_{t+1}^{-\gamma} \right]^{-\frac{1}{\gamma}} \tag{31}$$

which shows how current consumption depends on expectations of future values. Equation (26) describes the interest arbitrage condition and the forwarding-looking behavior of the exchange rate.

$$s_t = \mathbf{E}_t(s_{t+1}) - i_t + i_t^* + \theta_t \tag{32}$$

The above equations (27) and (28) also show that the solutions for Q_t^x and Q_t^m, which determine investment and the evolution of capital in each sector, come from forward-looking stochastic Euler equations.

$$Q_t^x = \mathbf{E}_t \left[\left(\frac{\vartheta_{t+1}}{\vartheta_t} \right) \left(\begin{array}{c} \Lambda_{t+1} \frac{P_{t+1}^x}{P_{t+1}} \left(A_{t+1}^x (1-\theta_x)(K_t^x)^{-\theta_x} + \frac{\phi_x \left(I_{t+1}^x\right)^2}{2(K_t^x)^2} \right) \\ +Q_{t+1}^x (1-\delta_x) \end{array} \right) \right] \tag{33}$$

$$Q_t^m = \mathbf{E}_t \left[\left(\frac{\vartheta_{t+1}}{\vartheta_t} \right) \left(\begin{array}{c} \Lambda_{t+1} \frac{P_{t+1}^m}{P_{t+1}} \left[A_{t+1}^m (1-\theta_m)(K_t^m)^{-\theta_m} + \frac{\phi_m \left(I_{t+1}^m\right)^2}{2(K_t^m)^2} \right] \\ +Q_{t+1}^m (1-\delta_m) \end{array} \right) \right] \tag{34}$$

The shadow price or replacement value of capital in each sector is equal to the discounted value of next period's marginal productivity, the adjustment costs due to the new capital stock, and the expected replacement value net of depreciation.

Thus the model has four "forward-looking" stochastic Euler equations, which determine C_t, s_t, Q_t^x, Q_t^m These variables, together with (24), in turn determine current investment I_t^x and I_t^m as describe by the conditions in (29) and (30).

2.1.5 Relative prices, exchange rate pass-through and stickiness

There are 7 prices (absolute and relative) to be determined (P^x, P^m, J, Z, P_t^f, P_t^n, P). The price of export goods is determined exogenously for a small open economy (P^{x*}) and its price in domestic currency is $P^x = SP^{x*}$. The price of import goods is also determined exogenously for a small open economy P^{m*}, but, we assume that price changes are incompletely passed-through (see Campa and Goldberg (2002) for a study on exchange rate pass-through and import prices). Using the definition: $P^m = SP^{m*}$ and assuming partial adjustment, we obtain:

$$p_t^m = \omega(s_t + p_t^{m*}) + (1-\omega) p_{t-1}^m \tag{35}$$

where $\omega = 1$ indicates complete pass-through of foreign price changes.

Thus, given P^x and P^m, we have $J = P^x/P^m$, and:

$$P_t^f = \left[(\alpha_x)^{-\alpha_x}(1-\alpha_x)^{-1+\alpha_x}\right](P_t^x)^{\alpha_x}(P_t^m)^{1-\alpha_x} \tag{36}$$

Finally, we obtain the aggregate consumption price deflator as:

$$P_t = \left[(\alpha_f)^{-\alpha_f}(1-\alpha_f)^{-1+\alpha_f}\right]\left(P_t^f\right)^{\alpha_f}(P_t^n)^{1-\alpha_f} \tag{37}$$

While the exchange rate is determined by the forward-looking interest parity relation, and the terms of trade are determined exogenously, the price of non-traded goods adjusts in response to demand and supply in this sector.

2.1.6 Macroeconomic Conditions And Market Clearing

The national accounting equation is:

$$\begin{aligned}
& P_t^x\left(Y_t^x - \frac{\phi_x}{2K_t^x}(I_t^x)^2\right) + P_t^m\left(Y_t^m - \frac{\phi_m}{2K_t^m}(I_t^m)^2\right) + P_t^n Y_t^n \\
= \; & P_t^x\left(C_t^x + X_t + I_t^x\right) + P_t^m\left(C_t^m - M_t + I_t^m\right) + P_t^n\left(C_t^n + G_t\right) \\
= \; & P_t C_t + \left(P_t^x I_t^x + P_t^m I_t^m\right) + \left(P_t^x X_t - P_t^m M_t\right) + P_t^n G_t \qquad (38)
\end{aligned}$$

Real gross domestic product is given as:

$$y = \frac{1}{P_t}\left[P_t^x\left(Y_t^x - \frac{\phi_x}{2K_t^x}(I_t^x)^2\right) + P_t^m\left(Y_t^m - \frac{\phi_m}{2K_t^m}(I_t^m)^2\right) + P_t^n Y_t^n\right] \tag{39}$$

The change in bond holdings and foreign debt holdings evolves as follows:

$$P_t^n G_t = B_{t+1} - B_t(1+i_t) \tag{40}$$

$$\left(P_t^x X_t - P_t^m M_t\right) = -S_t\left(L_{t+1}^* - L_t^*[1 + i_t^* + \theta_t]\right) \tag{41}$$

2.2 The Monetary Authority

The Central Bank adopts practices consistent with optimal control models, specifically, the linear quadratic regulator problem. It chooses an optimal interest rate reaction function, given its loss function equation, and its perception of the evolution of the state variables, inflation and growth. The change in the interest rate is the solution of the optimal linear quadratic regulator problem,with control variable Δi solved as a feedback response to the lagged state variables.

We assume, perhaps more realistically, that the monetary authority does not know the exact nature of the private sector model, instead it "learns" and updates the state-space model equation, which underpins its calculation of the optimal interest rate policy period by period. In other words, at each period time t, the Central Bank updates its information about the evolution of key economic variables, and re-estimates the state-space system to obtain new estimates. The central bank then uses this information to determine the optimal interest rate.

The central bank makes use of a linear quadratic loss function, and updates linear laws of motion of inflation and depreciation for finding its policy response. The response is a linear "reaction function". Admittedly this relatively simple linear quadratic framework of the central bank contrasts with the more complex nonlinear dynamic optimization process used by the private-sector decision makers. However, we note that the weights and coefficients of this linear quadratic framework are updated each period, and that the forecasting model used by the Central Bank generate inflation forecasts that do not deviate persistently from the underlying true inflation rates.

For this paper, two different policy scenarios are considered - a pure inflation targeting policy stance and an inflation-exchange rate policy stance. The weights for inflation and exchange rate depreciation in the loss function depend on the conditions at time t.

- Strict Inflation targeting

In the strict inflation target case, the monetary authority estimates or "learns" the evolution of inflation as a function of its own lag as well as of changes in the interest rate.

$$\Lambda_1 = \lambda_{1t}(\pi_t - \pi^*)^2 \tag{42}$$

$$x_t = \sum_{j=0}^{k} \Gamma_{1t,j} x_{t-j-1} + \Gamma_{2t} \Delta i_t + e_t \tag{43}$$

$$i_{t+1} = i_t + \sum_{j=0}^{k} h(\widehat{\Gamma}_{1t,j}, \widehat{\Gamma}_{2t}, \lambda_{1t}) x_{t-j} \tag{44}$$

where x_t contains the inflation variable $\pi_t = \log(P_t/P_{t-4})$, an annualized rate of inflation. π^* is the target for inflation, and k is the number of lags for forecasting the evolution of the state variable. The feedback function h is obtained by solving the linear quadratic regulator problem, as discussed in Sargent (1999).

The weight on the loss function, λ_{1t}, shown in Table 1, reflects the Central Bank's concerns about inflation and is dependent on the state of the economy.

Table I: Policy Weights	
Strict Inflation Targeting	
$\pi \leq \pi^*$	$\lambda_1 = 0.0$
$\pi > \pi^*$	$\lambda_1 = 1.0$

In this strict anti-inflation scenario, if inflation is less than the target level π^*, the central bank does not optimize; in other words, the interest rate remains at its level: $i_{t+1} = i_t$. This is the "no intervention" case. However, if inflation is above the target rate ($\pi > \pi^*$), the monetary authority implements its optimal interest policy according to equation (44).

- Flexible Inflation Targeting with Inflation and Exchange Rate Targets

In the flexible inflation targeting scenario, the central bank also considers the behavior of the exchange rate. It learns the evolution of inflation and exchange rate depreciations as functions of their own lags and of changes in the interest rate.

$$\Lambda_2 = \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\eta_t - \eta^*)^2 \quad (45)$$

$$x_t = \sum_{j=0}^{k} \Gamma_{1t,j} x_{t-j-1} + \Gamma_{2t} \Delta i_t + e_t \quad (46)$$

$$i_{t+1} = i_t + \sum_{j=0}^{k} h(\widehat{\Gamma}_{1t,j}, \widehat{\Gamma}_{2t}, \lambda_{1t}, \lambda_{2t}) x_{t-j} \quad (47)$$

where x_t contains the inflation variable π_t as well as the depreciation variable $n_t = \log(S_t/S_{t-4})$, the annualized rate of change of the exchange rate. The term η^* represents the target for exchange rate depreciation. In this case, we have a bivariate forecasting model for the evolution of the state variables, π_t and η_t, with an equal number of lags. The coefficient matrix $\Gamma_{1t,j}$, for k lags contains two $(k \times 1)$ recursively updated matrix coefficients, representing the effects of lagged inflation and depreciation on current inflation and depreciation. "Least squares learning" is used to forecast the future values of these "state" variables in each scenario.

The weights $(\lambda_{1t}, \lambda_{2t})$ reflects the flexible inflation targeting stance of the central bank. They are summarized in Table II.

Table II: Policy Weights		
Flexible Inflation Targeting		
	Exchange Rate Depreciation	
Inflation	$\eta_t \leq \eta^*$	$\eta_t > \eta^*$
$\pi \leq \pi^*$	$\lambda_1 = 0.0$	$\lambda_1 = 0.1$
	$\lambda_2 = 0.0$	$\lambda_2 = 0.9$
$\pi > \pi^*$	$\lambda_1 = 0.9$	$\lambda_1 = 0.5$
	$\lambda_2 = 0.1$	$\lambda_2 = 0.5$

In this policy scenario, if inflation is below the target level π^* and the change in the exchange rate is also below the target, η^*, then the central bank does not optimize and hence does not change the policy interest rate. If inflation is above the target rate ($\pi > \pi^*$), with within target exchange rate changes, ($\eta_t \leq \eta^*$) the monetary authority puts greater weight on inflation in its objective function, $\lambda_{1t} = 0.9$. In contrast, when the depreciation of the exchange rate is above target ($\eta_t > \eta^*$), with inflation below target ($\pi \leq \pi^*$), the exchange rate inflation weight dominates $\lambda_{2t} = 0.9$. Finally, if both inflation and depreciation are above their respective targets, the central bank puts equal weights on them in its objective function.

Thus, corresponding to each scenario, the central bank optimizes a loss function Λ with weights on the loss function, $\lambda_t = \{\lambda_{1t}, \lambda_{2t}\}$ dependent on the economic conditions at time t. The monetary authority estimates the state-space system and uses the parameter set $\widehat{\Gamma}_{1tj}$ to formulate an optimal interest-rate feedback rule according to equation (47) which is the solution of the optimal linear quadratic "regulator" problem, with control variable Δi solved as a feedback response to the state variables.

In formulating its optimal interest-rate feedback rule, the government acts at time t as if its estimated model for the evolution of the state variables are true "forever", and that its relative weights for inflation, or depreciation in the loss function are permanently fixed. However, as Sargent (1999) points out in a similar model, the monetary authority's own procedure for re-estimation "falsifies" this pretense as it updates the coefficients $\{\Gamma_{1tj}, \Gamma_{2t}\}$, and re-solves the linear quadratic regulator problem for a new optimal response "rule" of the interest rate with every bit of new information.

Sargent (1993) calls a system in which agents are "learning about a system that is being influenced by the learning process of people like themselves" a *self-referential* system. He notes that the dynamics induced by such a system are "transient", if the "adaptive algorithms" of the agents are "boundedly rational" [Sargent (1993), p. 132]. We show, below, that the learning behavior of the central bank is indeed "boundedly rational", so that the dynamics are indeed transient.

3 Calibration and Solution Algorithm

The section discusses the calibration of the parameters, the initial conditions, and the stochastic processes for the exogenous variables (P^{x*}, P^{m*}) and the risk premia (θ_t). It also contains a brief discussion of the parameterized expectations algorithm (PEA) for solving the model.

3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table III.

Table III: Calibrated Parameters	
Consumption:	$\gamma = 1.5,\ \ \beta = 0.009,\ \ \alpha_x = 0.5,\ \ \alpha_f = 0.5$
Production:	$\theta_m = 0.7,\ \ \theta_x = 0.3,\ \ \delta_x = \delta_m = 0.025,\ \ \phi_x = \phi_m = 0.03$

Many of the parameter selections follow Mendoza (1995). The constant relative risk aversion γ is set at 1.5 (to allow for high interest sensitivity).

The shares of non-traded goods in overall consumption is set at 0.5, while the shares of exports and imports in traded goods consumption is 50 percent each. Production in the export goods sector is more capital intensive than in the import goods sector.

The initial values of the nominal exchange rate, the price of non-tradeables and the price of importable and exportable goods are normalized at unity while the initial values for the stock of capital and financial assets (domestic and foreign debt) are selected so that they are compatible with the implied steady state value of consumption, $\overline{C} = 2.02$, which is given by the interest rate and the endogenous discount factor. The values of $\overline{C}^x$, $\overline{C}^m$, and $\overline{C}^n$ were calculated on the basis of the preference parameters in the sub-utility functions and the initial values of B and L^* deduced.

Similarly, the initial shadow price of capital for each sector is set at its steady state value. The production function coefficients A^m and A^x, along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector.

Finally, the foreign interest rate i^* is also fixed at the annual rate of 0.04. In the simulations, the effect of initialization is mitigated by discarding the first 15% of the simulated values.

3.2 Terms of Trade and Currency Risk

The terms of trade shocks are modelled as follows:

$$\begin{aligned} p_t^{x*} &= p_{t-1}^{x*} + \varepsilon_t^{x*}; \quad \varepsilon_t^{x*} \sim N(0, 0.01) \\ p_t^{m*} &= p_{t-1}^{m*} + \varepsilon_t^{m*}, \quad \varepsilon_t^{m*} \sim N(0, 0.01) \end{aligned}$$

where lower case denotes the logs of the respective prices. The evolution of the price variables p_t^{x*} and p_t^{m*} mimic actual data generating processes, namely that the variable is a unit-root autoregressive process, with a normally distributed innovation with standard deviation set at 0.01. The errors are assumed to be independent.

As noted earlier, the domestic price of export goods fully reflect the exogenously determined prices, but the domestic price of import goods are only partially passed on (see equation (35)) with ω as the coefficient of exchange rate pass-through. We consider two cases in the simulations - high pass-through ($\omega = 0.8$) and low pass-through ($\omega = 0.4$).

The parameter values for the evolution of currency risk appear in Table VII.

Table VII: Currency Risk Parameters
$\xi_0 = 0.0, \ \xi_2 = 0.7, \ \xi_3 = 0.2$

There is no constant "currency risk" ($\xi_0 = 0$). The other coefficients are chosen to reflect empirical evidence of a stronger lagged effect ($\xi_2 = 0.7$) and a lower but significant response to unexpected shocks ($\xi_3 = 0.2$). The shocks η_t are drawn from a t-distribution with 2 degrees of freedom, and with $\sigma_\eta = 0.1$. This mimics reality better and yields risk premia that have leptokurtic properties.

In brief, this is a simulation study about the design of monetary policy for an economy subjected to external shocks in the form of relative price shocks and currency risk shocks.

3.3 Solution Algorithm and Constraints

3.4 Solution Algorithm

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to parameterize the forward-looking expectations in this model, with non-linear functional forms:

$$
\begin{aligned}
E_t C_{t+1} &= \psi^C(\mathbf{x}_{t-1}; \Omega_C) &(48)\\
E_t s_{t+1} &= \psi^S(\mathbf{x}_{t-1}; \Omega_S) &(49)\\
E_t Q^x_{t+1} &= \psi^{Q^x}(\mathbf{x}_{t-1}; \Omega_{Q^x}) &(50)\\
E_t Q^m_{t+1} &= \psi^{Q^m}(\mathbf{x}_{t-1}; \Omega_{Q^m}) &(51)
\end{aligned}
$$

The vector $\mathbf{x}_{t-1}$ contains a set of observable instrumental variables at time t and they are: consumption of import C^m and export goods C^x, the marginal utility of consumption λ, the real interest rate r, the real exchange rate, Z, and the shadow prices of replacement capital for the two sectors, Q^m and Q^x, all expressed in deviations from their initial steady state:

$$
\mathbf{x}_{t-1} = \{C^m - \overline{C^m}, C^x - \overline{C^x}, \lambda - \overline{\lambda}, r - \overline{r}, Z - \overline{Z}, Q^m - \overline{Q^m}, Q^x - \overline{Q^x}\} \quad (52)
$$

The symbols $\Omega_\lambda, \Omega_S, \Omega_{Q^x}$, and Ω_{Q^m} represent the parameters for the expectation function, while $\psi^C, \psi^S, \psi^{Q^x}$ and ψ^{Qf} are the expectation approximation functions.

Judd (1996) classifies this approach as a "projection" or a "weighted residual" method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). The functional forms for ψ^S, ψ^C, ψ^{Q^x}, ψ^{Qf} are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks can produce results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation. Judd (1996) notes that the neural networks provide us with an "inherently nonlinear functional form" for approximation, in contrast with methods based on linear combinations of polynomial and trigonometric functions. Both Judd (1996) and Sargent (1997) have drawn attention to the work of Barron (1993), who found that neural networks do a better job of "approximating" any non-linear function than polynomials, in the sense that a neural network achieves the same degree of in-sample predictive accuracy with fewer parameters, or achieves greater accuracy, using the same number of parameters. For this reason, the approach of this study uses neural networks as the approximation functions.

Since the parameterized expectation equations are relatively complex non-linear functions, the optimization problem is solved with a repeated hybrid approach. First a global search method, genetic algorithm,[7] similar to

[7] De Falco (1998) applied the genetic algorithm to nonlinear neural network estimation,

the one developed by Duffy and McNelis (2001), is used to find the initial parameter set, $(\Omega_\lambda, \Omega_S, \Omega_{Q^x}, \Omega_{Q^m})$. These parameters together with $\mathbf{x}_{t-1}$ are then used to determine the expectational variables identified in equations (48)-(51). The entire model is then solved to yield values for all the endogenous variables and expectational errors for the four forward looking variables are computed. The procedure is repeated for another set of values for $\{\Omega_C, \Omega_S, \Omega_{Q^x}, \Omega_{Q^m}\}$ and convergence is obtained when the expectational errors are minimized. In the repeated simulations, a local optimization, the BFGS method, based on the quasi-Newton algorithm, is used to "fine tune" the genetic algorithm solution.[8] In short, the solution algorithm for parameterized expectations makes use of neural network specification for the expectations functions, and a genetic algorithm for the iterative solution method, as well as the quasi-Newton method.

In the algorithm, the following non-negativity constraints for consumption and the stocks of capital were imposed:

$$C_t^x > 0, \quad K_t^x > 0, \quad K_t^m > 0 \tag{53}$$

The latter was achieved by assuming irreversible investment for capital in each sector, that is for $i = X, M$:

$$I_t^i = \begin{cases} \frac{1}{\phi_i}\left(\frac{P_t}{P_t^x}\frac{Q_t^i}{\Lambda_t} - 1\right) K_{t-1}^i & \text{if } \frac{P_t}{P_t^x}\frac{Q_t^i}{\Lambda_t} > 1 \\ 0 & otherwise \end{cases} \tag{54}$$

For domestic debt, we assume that sufficient tax is levied each period to service the debt, that is $T_t = i_{t-1}B_{t-1}$, which implies that $B_t = B_{t-1}(1 + i_{t-1}) - T_t + P_t^n G_t$. Then with $G = 0$, B remains unchanged in this study.

The usual no-Ponzi game applies to the evolution of foreign assets and we fulfill the transversality condition by keeping the foreign debt to GDP

and found that his results "proved the effectiveness" of such algorithms for neural network estimation. The main drawback of the genetic algorithm is that it is slow. For even a reasonable size or dimension of the coefficient vector, the various combinations and permutations of the coefficients which the genetic search may find "optimal" or close to optimal, at various generations, may become very large. This is another example of the well-known "curse of dimensionality" in non-linear optimization. Thus, one needs to let the genetic algorithm "run" over a large number of generations—perhaps several hundred—in order to arrive at results which resemble unique and global minimum points.

[8] Quagliarella and Vicini (1998) point out that hybridization may lead to better solutions than those obtainable using the two methods individually. They argue that it is not necessary to carry out the quasi-Newton optimization until convergence, if one is going to repeat the process several times. The utility of the quasi-Newton BFGS algorithm is its ability to improve the "individuals it treats", so "its beneficial effects can be obtained just performing a few iterations each time" [Quagliarella and Vicini (1998): 307].

ratio bounded by imposing the following constraints on the parameterized expectations algorithm[9]

$$\left(\frac{|S_t L_t^*|}{P_t y_t}\right) < \widetilde{L} \tag{55}$$

where $\widetilde{L}$ is the critical foreign debt ratio. If the external debt grows above a critical external debt/gdp ratio, $\widetilde{L^*}$, the fiscal authority will levy taxes in the traded-goods sector in order to reduce or buy-back external debt.

4 Simulation Analysis

4.1 Base-Line Results

The aim of the simulations is to compare the outcomes for inflation, growth and welfare for the two policy scenarios - strict inflation policy (targeting π only) and flexible inflation policy (targeting inflation π and depreciation η) under two sticky-price scenarios - high pass-through ($\omega = 0.8$) and low pass-through ($\omega = 0.4$). To ensure that the results are robust, we conducted 1000 simulations (each containing a time-series of 220 realizations of terms of trade and currency risk shocks).

Figure 1 shows the simulated paths for one time series realization of the exogenous terms of trade index as well as the time-varying currency risk. The terms of trade shocks reflect reality in that there were periods of improvements (upward trend) and periods of deteriorations (downward trend). This particular realization of the currency risk variable also captures the typical pattern of periods of low risk followed by periods of unexpectedly high risks.

The simulated values for the key variables (inflation, growth, real exchange rate, current account) are well-behaved. Figure 2 presents the evolution of consumption for the 2 pass-through cases under the two policy scenarios. As shown, in general, despite the large swings in the terms of trade index, consumption does not deviate appreciably and for long periods from its steady-state value. Also in general, the adjustment of consumption is less volatile with the introduction of multiple targets for the monetary authority and more volatile the higher the pass-through (i.e., less price stickiness).

To ascertain which policy regime yields the higher welfare value, we examined the distribution of the welfare outcomes of the different policy regimes

[9] In the PEA algorithm, the error function will be penalized if the foreign debt/GDP ratio is violated. Thus, the coefficients for the optimal decision rules will yield debt/GDP ratios which are well below levels at which the constraint becomes binding.

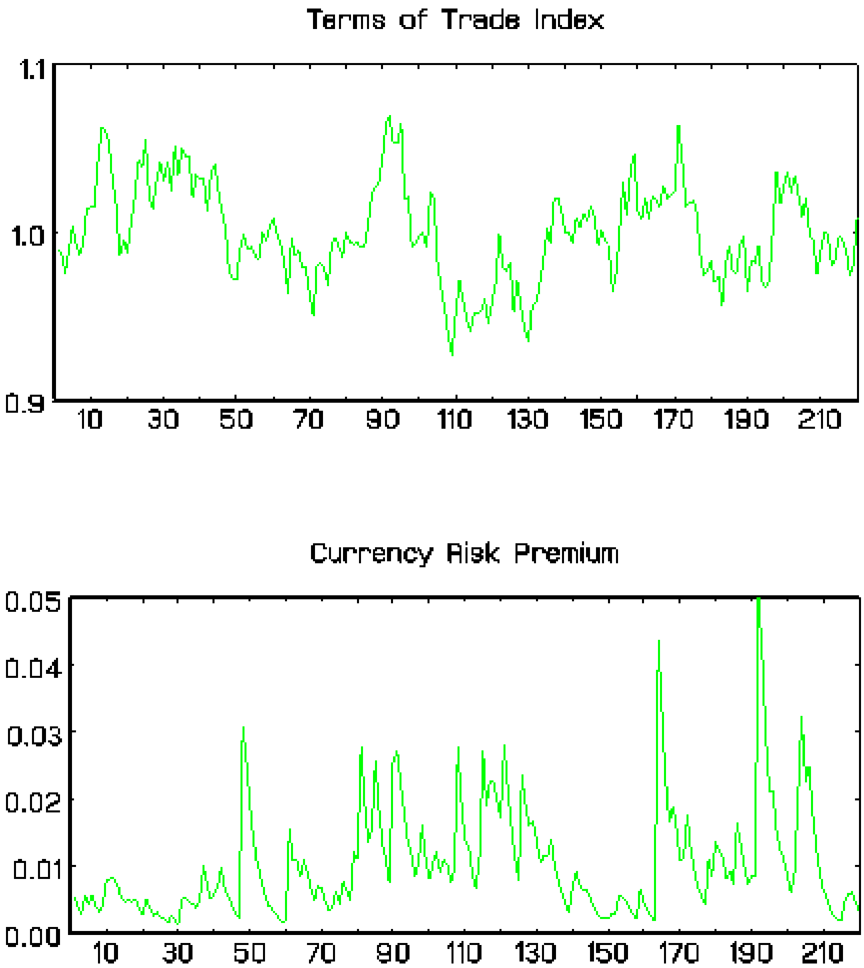

Figure 1: Shock Processes

for 1000 different realizations of the terms of trade and risk premia shocks. Before presenting these results, we evaluated the accuracy of the simulation results as well as the rationality of the learning mechanism.

4.2 Den Haan-Marcet Accuracy Test

The accuracy of the simulations is checked by the Den Haan-Marcet statistic, originally developed for the parameterized expectations solution algorithm but applicable to other procedures as well. This test makes use of the Euler equation for consumption, under the assumption that with accurate expectations, the path of consumption would be optimal, so that expectational term in the Euler equation may be replaced by the actual term and a random error term, ν_t :

$$\vartheta_{t+1}\Lambda_{t+1}\frac{1}{P_{t+1}}(1+i_t) - \vartheta_t\Lambda_t\frac{1}{P_t} = \nu_t$$

To test whether ν_t is significantly different from zero, Den Haan and Marcet propose a transformation of ν_t which has a chi-squared distribution under the hypothesis of accuracy. If the value of this statistic belongs to the upper or lower critical region of the chi-squared distribution, Den Haan and Marcet suggest that this is evidence "against the accuracy of the solution". [Den Haan and Marcet (1994): p. 5].

Table IV presents the percentage of realizations (out of 1000) in which the Den Haan-Marcet statistics fell in the upper or lower critical regions of the chi-squared distribution, for each policy regime, under alternative pass-through coefficients. As shown, the simulated results may be deemed to be accurate.

Table IV: Distribution of Den-Haan Marcet Statistic
Percentage in Upper/Lower Critical Region

	pass-through coefficient	
Policy Regime	$\omega = 0.8$	$\omega = 0.4$
Targeting π	5.4/0.6	2.6/3.9
Targeting π, η	4.8/0.4	3.0/10.6

4.3 Learning and Quasi-Rationality

In our model, the central bank learns the underlying process for inflation in the strict inflation targeting regime and the underlying processes for inflation and depreciation under the flexible inflation targeting regime. The learning takes place by updating recursively the least-squares estimates of a vector

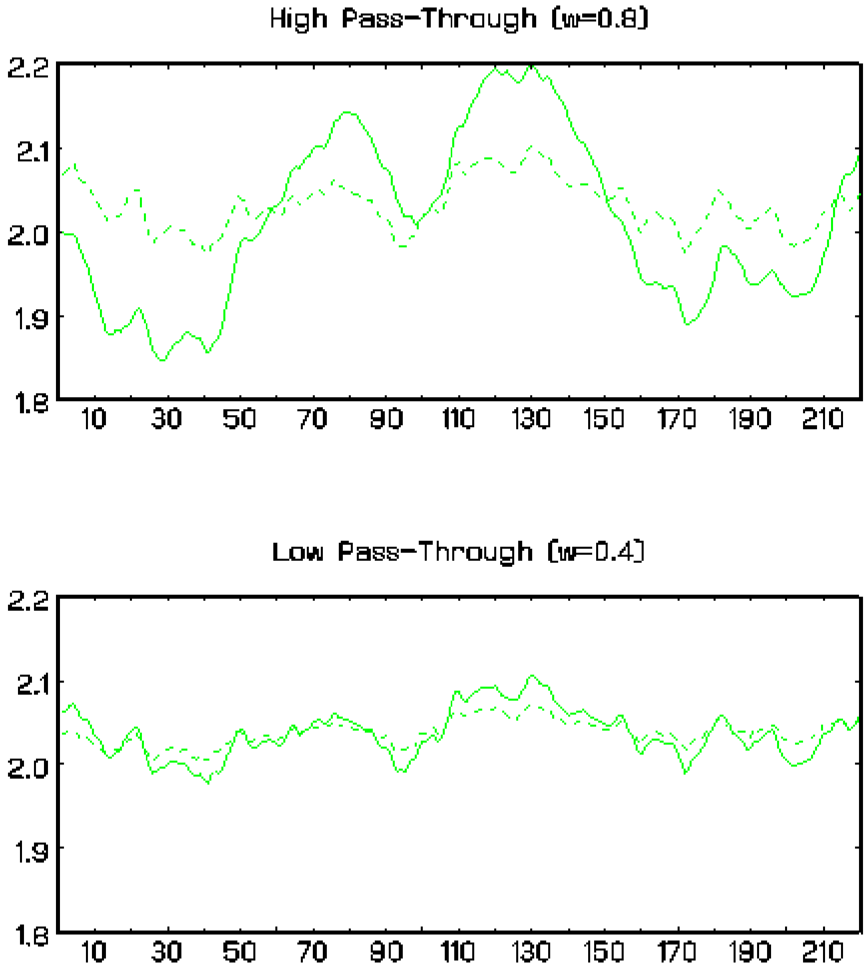

Figure 2: Consumption: solid line (strict inflation targeting); dotted line (flexible inflation targeting)

autoregressive model. As we mentioned above, the dynamics induced by this learning behavior are "transient dynamics" if the agents are "boundedly rational".

Marcet and Nicolini (1997) raise the issue of reasonable rationality requirements in their discussion of recurrent hyperinflations and learning behavior. In our model, a similar issue arises. Given that the only shocks in the model are recurring shocks, with no abrupt, unexpected structural changes taking place, the learning behavior of the central bank should not depart, for too long, from the rational expectations paths. The central bank, after a certain period of time, should develop forecasts which converge to the true inflation and exchange rate depreciation paths of the economy, unless we wish to make some special assumption about monetary authority behavior.

Marcet and Nicolini discuss the concepts of "asymptotic rationality", "epsilon-delta rationality" and "internal consistency", as criteria for "boundedly rational" solutions. They draw attention to the work of Bray and Savin (1996). These authors examine whether the learning model rejects serially uncorrelated prediction errors between the learning model and the rational expectations solution, with the use of the Durbin-Watson statistic. Marcet and Nicolini point out that the Bray-Savin method carries the flavor of "epsilon-delta" rationality in the sense that it requires that the learning schemes be consistent "even along the transition" [Marcet and Nicolini (1997): p.16, footnote 22].

Following Bray and Savin, we use the Durbin-Watson statistic to examine whether the learning behavior is "boundedly rational". Table V gives the Durbin-Watson statistics for the inflation forecast errors of the central bank, under both policy regimes, and under alternative pass-through coefficients. In all but one case, we see that the learning behavior does not violate the requirements of bounded rationality (the percentage is below 5%). In the case of flexible inflation targeting with $\omega = 0.4$, there is evidence that learning is not bounded rationality, in other words, the implied evolution of exchange rate changes is not easily captured by our vector-autoregressive model.

Table V: Durbin-Watson Statistics for Forecast Errors
Percentage in Lower and Upper Critical Region

	$\omega = 0.8$		$\omega = 0.4$	
Policy Regime	Inflation	Depreciation	Inflation	Depreciation
Inflation π	0.0/0.0	—	0.2/0.0	—
Targeting π, η	0.0/0.2	3.4/0.1	1.1/0.0	13.1/0.0

4.4 Comparative Welfare Results

This section summarizes the results for 1000 alternate realizations of the terms-of-trade and currency risk shocks (each realization contains 220 observations). For each realization we compute the mean welfare and the 1000 sample means are presented in a distributional form. Figure 3 (upper panel) shows that there is indeed a reduction in the *spread* of welfare as the central bank changes its targets from the strict inflation targeting scenario to the flexible inflation/depreciation scenario. However, the distributions in the two cases indicate different outcomes. In the case of high pass through, strict inflation targeting may lead to lower welfare levels, whereas in the case of lower pass through, it may lead to decidedly higher levels of welfare.

4.5 Relation to Optimal Monetary Policy

To better assess our results, we compare the welfare estimates obtained under learning with those given by optimal monetary policy. The optimal monetary policy we take as the benchmark is the use of a Taylor rule, in which the change in the interest rate, $i_{t+1}-i_t$, responds to the difference between actual inflation, π_t,and target inflation, π^*, as well as between actual depreciation rate, η_t, and target rate, η^* :

$$i_{t+1} - i_t = \lambda_\pi(\pi_t - \pi^*) + \lambda_\eta(\eta_t - \eta^*) \tag{56}$$

In this case, the policy coefficients λ_π and λ_η are estimated along with the other PEA coefficients yielding the optimal decision rules for consumption, for the expected exchange rate, and the expected values of Tobin's Q, through the parameterized expectations algorithm. In other words, we have an "altruistic" monetary authority, interested in optimizing the intertemporal welfare of the representative consumer, by adjusting its interest rate rule in response to deviations of inflation and depreciation from their target levels.

We compare the welfare outcomes under learning with two versions of the optimal Taylor rule: one in which $\lambda_\pi \neq 0$, $\lambda_\eta = 0$, i.e., a restricted Taylor rule, in which interest rates only respond to inflation targets; and one in which $\lambda_\pi \neq 0$, $\lambda_\eta \neq 0$, i.e, an unrestricted Taylor rule, in which interest rates respond to both inflation and depreciation targets. While this very simplified Taylor rule is only one among many for the computation of optimal monetary policy, it serves as a ready benchmark for determining how well the learning process and recursive updating of feedback coefficients converge to the welfare obtained by an optimal monetary policy framework where the monetary authorities act to maximize overall welfare.

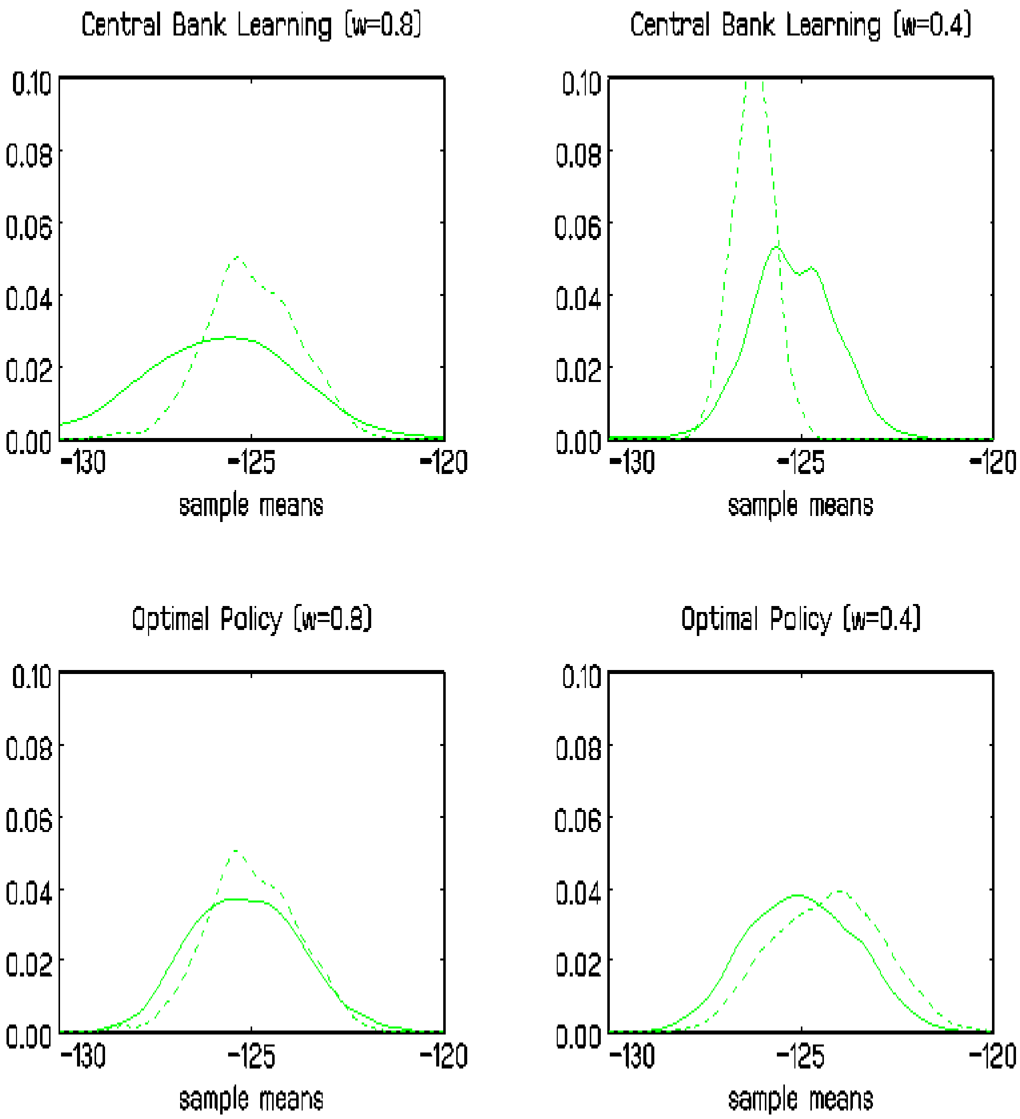

Figure 3: Distributions of the Welfare Index: solid line (strict inflation targeting); dotted line (flexible inflation targeting)

Figure 3 (bottom panel) pictures the distributions of the two optimal Taylor-type feedback monetary policy rules. A number of conclusions emerge. One is that under high pass through (and thus high price flexibility), as well as relatively low pass-through, it does not matter which monetary policy rule is used. The results are virtually identical. Optimal monetary policy rules, based on simple Taylor formulations, in our model, without any learning, do not give much support for flexible inflation targeting.

There is one other conclusion which comes from a comparison of the welfare distributions in Figure 3. It is clear that "stickiness" in information through central bank learning lowers welfare as in all comparable scenarios, the mean welfare under optimality dominates the mean welfare with central bank learning.[10] However, the overall welfare distributions obtained under learning with high pass-through are not markedly worse than the welfare distributions generated by optimal monetary policy with the simple Taylor framework. This indicates that the learning mechanism, with the continuous updating of the laws of motion of inflation and depreciation dynamics, as well as the revision of the interest-rate rule, is approximating an optimal Taylor framework. Central bank learning rules can bring the economy to welfare outcomes which are close to the simplified optimal Taylor rules. In general, overall welfare falls as the degree of price stickiness increases, and the number of the variables the central bank has to learn increases - the lowest mean welfare in these eight scenarios is when $\omega = 0.4$ and when the central bank is targeting and learning both inflation and depreciation.

5 Conclusions

This paper has compared two alternative policy scenarios - strict and flexible inflation targeting - for an economy facing terms of trade shocks and time-varying currency risk. The central bank is also assumed not to have full information about all behavioral aspects of the economy, instead it has to learn the laws of motion for its key target variables in order to set the interest rate according to a feedback rule.

The results show that including exchange rate changes in its learning and policy targeting framework improves welfare under the case of very high exchange-rate pass-through in the sense that the probability of low welfare

[10] In may be argued that central banks have more sophisticated knowledge of underlying inflation dynamics than that which is implied by linear least squares learning. However, as shown here, linear least squares learning can be a good “tracking” mechanism for more complex dynamic processes and the recursive method describe here serves as an approximation to the Kalman filtering method for updating and learning.

outcomes is reduced. However, if "information" becomes more transparent, so that policy-makers can implement optimal welfare-maximizing rules, there is little or no case for including exchange-rate depreciation targets. The policy implication is that central banks which are already targeting inflation, should resist pressures to adopt exchange-rate targets. This implication is particular significant the greater the degree of price stickiness in the system.

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