# Inflation Targeting, Learning and Q Volatility in Small Open Economies

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**Abstract**

This paper examines the welfare implications of managing asset-price with consumer-price inflation targeting by monetary authorities who have to learn the laws of motion for both inflation rates. The central bank can reduce the volatility of consumption as well as improve welfare more effectively if it adopts state-contingent Taylor rules aimed at inflation and Q-growth targets in this environment. Under perfect foresight, however, pure inflation targets dominate combined consumer and asset-price inflation targets.

*Key words*: Tobin's Q, learning, Taylor policy rules, inflation targets

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# 1 Introduction

Many countries now practice inflation targeting, but that has not immunized the economy from experiencing asset price volatility in the form of exchange rate instability in Australia or share-market bubbles in the United States. The practice of controlling changes in goods prices is taken for granted by many Central Banks, but there is no consensus about the management of asset-price inflation, except in the sense that it is not desirable for asset prices to be too high or too volatile. At the World Economic Forum in Davos in 2003, Lawrence Summers suggested that policy makers should use other tools, such as margin lending requirements or public jawboning, to combat asset-price inflation. He compared raising interest rates to combat asset-price inflation to a preemptive attack, and stated "it takes enormous hubris to know when the right moment has come to start a war" [Summers (2003), p.1].

Recent research shows that central bankers should not target asset prices [see. for example. Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) for a closed economy study]. However, Cecchetti, Genberg and Wadhwani (2002) have argued that central banks should "react to asset price misalignments". In essence, they show that when disturbances are nominal, reacting to close misalignment gaps significantly improves macroeconomic performance. Smets (1997) has also stressed that the proper response of monetary policy to asset-price inflation depends on the source of the asset-price movements. If productivity changes are the driving force, accommodation is called for, and real interest rates should remain unchanged. However, if the source is due to non-fundamental shocks in the equity market, in the form of bullish predictions about productivity, then monetary policy should raise interest rates.

In contrast to previous studies we evaluate monetary policy in a small open-economy framework, and in particular we are concerned with investment in a resource-rich small open economy subjected to the vagaries of international terms of trade shocks. Detken and Smets (2004) have shown that high cost asset-price booms are as common in small open economies subject to fundamental terms-of-trade shocks as they are in relatively closed economies driven by fundamental productivity shocks.

We also highlight learning on the part of the Central Bank. For a small open economy subject to terms of trade movements, learning behavior on the part of the policy authority is an appropriate assumption, since movements of the terms of trade are determined in international markets far removed from the influence of domestic policy actions. In this context, central banks are more likely to be engaged in learning behavior.[1]

The economy we study has an export sector and an imported manufactured goods sectors. The terms of trade are driven by movements in the commodity export price relative to the price of manufactured goods. The volatility of this relative price in turn affects share prices and investment in the booming (or declining) export sector.

In this paper, we consider the rate of growth of Tobin's Q, first introduced by Tobin (1969), as a potential target variable for monetary policy. Our reasoning is that Q-growth would be small when the growth in the market valuation of capital assets corresponds roughly with the growth of replacement costs. Since asset prices (in the market value) are a lot less sticky than good prices (in the replacement cost), the presence of high Q-growth would be indicative of misalignment of market value and replacement cost, in other words an indication of an "excessive" change in the share price. Thus monitoring and targeting Q-growth may be viewed as a proxy policy for monitoring and targeting asset price inflation, but with the advantage that the asset price is evaluated relative to a benchmark (the replacement cost).

The focus on Q is also influenced by Brainard and Tobin (1977), who argued that Q plays an important role in the transmission of monetary policy both directly via the capital investment decision of enterprises and indirectly via consumption decisions. Thus volatility of Q has implications for inflation and growth. Large swings in Q can lead to systematic over-investment, and in the open-economy context, over-borrowing and serious capital account deficits.

This paper is concerned with the thought experiment: what happens to consumption, inflation and welfare if the central bank also monitors Q? In particular, we will generate the welfare implications of adopting a stance of monetary policy which includes targeting consumer price inflation as well as changes in Q. We assume that the policy makers has to learn about the nature of the shock as well as the underlying laws of motion of Q-growth and price inflation, subject to uncertainty about the underlying model.

We show in this paper that learning is the key assumption for justifying asset-price inflation targeting. If the central bank knows the true model as well as the shocks driving the economy, there is little reason to target asset-price inflation. Since the underlying true inflation process depends, in part, on asset-price inflation, there is no need to target this extra variable. However, if the central bank has to learn and forecast inflation, then including

[1] See Bullard and Metra (2002) and Evans and Honkapohja (2003) for studies with private sector learning.

asset-price inflation improves its forecasts of inflation, and turns out to be welfare enhancing.

We first examine the performance of optimal Taylor rules when the central bank knows the underlying true model as well as the shocks driving the terms of trade. We show that there is little case for including asset-price inflation as a target for monetary policy. Optimal welfare-maximizing Taylor rules without asset-price inflation have little or no interest-rate smoothing. However, if asset-price inflation is included, the degree of interest-rate smoothing increases. But the welfare differences are minor.

We then present the implications for two monetary policy scenarios with learning behavior. First, we consider standard Taylor rules for inflation targeting with and without reacting to Q-growth and then we examine state-contingent Taylor rules where monetary policy is more cautious. In this case, policy makers react to price inflation or Q-growth only when their forecasts cross critical thresholds, otherwise they refrain from taking action by raising or lowering interest rates, except in a worst-case scenario. This approach is similar to a "worst-case" robust control approach to monetary policy design, put forward by Rustem, Wieland, and Žaković (2005). They show that under uncertainty this approach leads to more moderate policy responses and represents a form of "cautionary monetary policy" advocated by Brainard (1967), who argued that the degree of policy activism should vary inversely with the extent of uncertainty about policy effectiveness [Rustem, Wieland, and Žaković (2005): p. 15]. The policy framework is also in line with Gruen, Plumb, and Stone (2005) who advocated a robust approach when information about the nature of the bubble is unavailable to the policy authority.

The paper is organized as follows. The model is described in Section 2, and the solution algorithm is presented in Section 3. Section 4 contains the simulation results for the alternative policy frameworks with and without learning. Concluding remarks are in Section 5.

# 2 Model Specification

The framework of analysis contains two modules - a module which describes the behavior of the private sector and a module which describes the behavior of the central bank.

## 2.1 Private Sector Behavior

The private sector is assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models.

### 2.1.1 Consumption

The utility function for the private sector "representative agent" is given by the following function:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{1}$$

where $C$ is the aggregate consumption index and $\gamma$ is the coefficient of relative risk aversion. Unless otherwise specified, upper case variables denote the levels of the variables while lower case letters denote logarithms of the same variables. The exception is the nominal interest rate denoted as $i$.

The representative agent as "household/firm"[2] optimizes the following intertemporal welfare function, with an endogenous discount factor:

$$W_t = \mathbf{E}_t \left[ \sum_{i=0}^{\infty} \vartheta_{t+i} U(C_{t+i}) \right] \tag{2}$$

$$\vartheta_{t+1+i} = [1 + \overline{C}_t]^{-\beta} \cdot \vartheta_{t+i}; \qquad \vartheta_t = 1 \tag{3}$$

where $\mathbf{E}_t$ is the expectations operator, conditional on information available at time $t$, while $\beta$ approximates the elasticity of the endogenous discount factor $\vartheta$ with respect to the average consumption index, $\overline{C}$. Endogenous discounting is due to Uzawa (1968) and is needed for the model to produce well-behaved dynamics with deterministic stationary equilibria.[3]

The specification used in this paper is due to Schmitt-Grohé and Uribe (2003). In our model, an individual agent's discount factor does not depend on their own consumption, but rather their discount factor depends on the average level of consumption. Schmitt-Grohé and Uribe (2003) argue that this simplification reduces the equilibrium conditions by one Euler equation and one state variable, over the standard model with endogenous discounting, it greatly facilitates the computation of the equilibrium dynamics, while delivering "virtually identical" predictions of key macroeconomic variables as the standard endogenous-discounting model.[4]

The consumption index is a composite index of non-tradeable goods $n$ and tradeable goods $f$ :

$$C_t = \left(C_t^f\right)^{\alpha_f} (C_t^n)^{1-\alpha_f} \tag{4}$$

[2] There is also no explicit government sector, taxes or bonds in this model.

[3] Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

[4] Schmitt-Grohé and Uribe (2003) argue that if the reason for introducing endogenous discounting is solely for introducing stationarity, "computational convenience" should be the decisive factor for modifying the standard Uzawa-type model.

where $\alpha_f$ is the proportion of traded goods. Given the aggregate consumption expenditure constraint,

$$P_t C_t = P_t^f C_t^f + P_t^n C_t^n \tag{5}$$

and the definition of the real exchange rate,

$$Z_t = \frac{P_t^f}{P_t^n} \tag{6}$$

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate $Z$:

$$C_t^f = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{-1+\alpha_f} Z_t^{-1+\alpha_f} C_t \tag{7}$$

$$C_t^n = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{\alpha_f} Z_t^{\alpha_f} C_t \tag{8}$$

Similarly, we can express the consumption of traded goods as a composite index of the consumption of export goods, $C^x$, and import goods $C^m$:

$$C_t^f = (C_t^x)_t^{\alpha_x} (C_t^m)^{1-\alpha_x} \tag{9}$$

where $\alpha_x$ is the proportion of export goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

$$P_t^f C_t^f = P_t^m C_t^m + P_t^x C_t^x \tag{10}$$

where $P^x$ and $P^m$ are the prices of export and import type goods respectively. Defining the terms of trade index $J$ as:

$$J = \frac{P^x}{P^m} \tag{11}$$

yields the demand for export and import goods as functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$C_t^x = \left(\frac{1-\alpha_x}{\alpha_x}\right)^{-1+\alpha_x} J_t^{-1+\alpha_x} C_t^f \tag{12}$$

$$C_t^m = \left(\frac{1-\alpha_x}{\alpha_x}\right)^{\alpha_x} J_t^{\alpha_x} C_t^f \tag{13}$$

### 2.1.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$Y_t^x = A_t^x (K_{t-1}^x)^{1-\theta_x} \tag{14}$$
$$Y_t^m = A_t^m (K_{t-1}^m)^{1-\theta_m} \tag{15}$$

where $A^x, A^m$ represents the labour factor productivity terms[5] in the production of export and import goods, and $(1-\theta_x)$, $(1-\theta_m)$ are the coefficients of the capital $K^x$ and $K^m$ respectively. The time subscripts $(t-1)$ indicates that they are the beginning-of-period values. The production of non-traded goods, which is usually in services, is given by the labour productivity term, $A_t^n$ :

$$Y_t^n = A_t^n \tag{16}$$

Capital in each sector has the respective depreciation rates, $\delta_x$ and $\delta_m$, and evolves according to the following identities:

$$K_t^x = (1-\delta_x)K_{t-1}^x + I_t^x \tag{17}$$
$$K_t^m = (1-\delta_m)K_{t-1}^m + I_t^m \tag{18}$$

where $I_t^x$ and $I_t^m$ represents investment in each sector.

### 2.1.3 Budget Constraint

The budget constraint faced by the household/firm representative agent is:

$$P_t C_t = \Pi_t + S_t \left[L_t^* - L_{t-1}^*(1+i_{t-1}^*)\right] - \left[B_t - B_{t-1}(1+i_{t-1})\right] - Tax_t \tag{19}$$

where $S$ is the exchange rate (defined as domestic currency per foreign), $L_t^*$ is foreign debt in foreign currency, and $B_t$ is domestic debt in domestic currency and $Tax_t$ is a lump sum tax. Profits $\Pi$ is defined by the following expression:

$$\begin{aligned}\Pi_t = {} & P_t^x \left[A_t^x \left(K_{t-1}^x\right)^{1-\theta_x} - \frac{\phi_x}{2K_{t-1}^x}\left(I_t^x\right)^2 - I_t^x\right] \\ & + P_t^m \left[A_t^m (K_{t-1}^m)^{1-\theta_m} - \frac{\phi_m}{2K_{t-1}^m}\left(I_t^m\right)^2 - I_t^m\right] + P_t^n A_t^n \end{aligned} \tag{20}$$

The aggregate resource constraint shows that the firm faces quadratic adjustment costs when they accumulate capital, with these costs given by the terms $\frac{\phi_x}{2K_{t-1}^x}\left(I_t^x\right)^2$ and $\frac{\phi_m}{2K_{t-1}^m}\left(I_t^m\right)^2$.

[5] Since the representative agent determines both consumption and production decisions, we have simplified the analysis by abstracting from issues about labour-leisure choice and wage determination.

The household/firm may lend to the domestic government and accumulate bonds $B$ which pay the nominal interest rate $i$. They can also borrow internationally and accumulate international debt $L^*$ at the fixed rate $i^*$, but this would also include a cost of currency exchange.[6]

The bond holdings and foreign debt holdings evolve as follows:

$$B_{t+1} = B_t(1+i_t) - Tax_t + P_t^n G_t \tag{21}$$

$$S_t L_{t+1}^* = S_t L_t^*(1+i_t^*) + (P_t^m M_t - P_t^x X_t) \tag{22}$$

where $G$ is government expenditure (exogenously determined).

### 2.1.4 Euler Equations

The household/firm optimizes the expected value of the utility of consumption (2) subject to the budget constraint defined in (19) and (20) and the constraints in (17) and (18).

$$\begin{aligned}
Max \quad : \quad & \text{Ł} = \mathbf{E}_t \sum_{i=0}^{\infty} \vartheta_{t+i} \{ U(C_{t+i}) \\
& -\Lambda_{t+i}[C_{t+i} - \frac{P_{t+i}^x}{P_{t+i}} \left( A_{t+i}^x (K_{t-1+i}^x)^{1-\theta_x} - \frac{\phi_x}{2K_{t-1+i}^x} \left(I_{t+i}^x\right)^2 - I_{t+i}^x \right) \\
& -\frac{P_{t+i}^m}{P_{t+i}} \left( A_{t+i}^m (K_{t-1+i}^m)^{1-\theta_m} - \frac{\phi_m}{2K_{t-1+i}^m} \left(I_{t+i}^m\right)^2 - I_{t+i}^m \right) - \frac{P_{t+i}^n}{P_{t+i}} A_{t+i}^n \\
& -\frac{S_{t+i}}{P_{t+i}} \left( L_{t+i}^* - L_{t-1+i}^*(1+i_{t-1+i}^*) \right) + \frac{1}{P_{t+i}} \left( B_{t+i} - B_{t-1+i}(1+i_{t-1+i}) \right) + Tax_t] \\
& -Q_{t+i}^x \left[ K_{t+i}^x - I_{t+i}^x - (1-\delta_x) K_{t-1+i}^x \right] \\
& -Q_{t+i}^m \left[ K_{t+i}^m - I_{t+i}^m - (1-\delta_m) K_{t-1+i}^m \right] \}
\end{aligned}$$

The variable $\Lambda$ is the familiar Lagrangian multiplier representing the marginal utility of wealth. The terms $Q^x$ and $Q^m$, known as Tobin's Q, represent the Lagrange multipliers for the evolution of capital in each sector - they are the "shadow prices" for new capital. Maximizing the Lagrangian with respect to $C_t, L_t^*, B_t, K_t^x, K_t^m, I_t^x, I_t^m$ yields the following first order conditions:

[6] The time-varying risk premium is assumed to be zero.

$$\Lambda_t = U'(C_t) \tag{23}$$

$$\vartheta_t U'(C_t)/P_t = \mathbf{E}_t \vartheta_{t+1} U'(C_{t+1})(1+i_t)/P_{t+1} \tag{24}$$

$$\vartheta_t U'(C_t) S_t/P_t = \mathbf{E}_t \vartheta_{t+1} U'(C_{t+1})(1+i_t^*) S_{t+1}/P_{t+1} \tag{25}$$

$$\left[\vartheta_t Q_t^x - \mathbf{E}_t \vartheta_{t+1} Q_{t+1}^x (1-\delta_x)\right] = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^x}{P_{t+1}} \left[ A_{t+1}^x (1-\theta_x)(K_t^x)^{-\theta_x} + \frac{\phi_x \left(I_{t+1}^x\right)^2}{2\left(K_t^x\right)^2} \right] \tag{26}$$

$$\left[\vartheta_t Q_t^m - \mathbf{E}_t \vartheta_{t+1} Q_{t+1}^m (1-\delta_m)\right] = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^m}{P_{t+1}} \left[ A_{t+1}^m (1-\theta_m)(K_t^m)^{-\theta_m} + \frac{\phi_m \left(I_{t+1}^m\right)^2}{2\left(K_t^m\right)^2} \right] \tag{27}$$

$$I_t^x = \frac{1}{\phi_x}\left(\frac{Q_t^x}{\Lambda_t} - 1\right) K_{t-1}^x \tag{28}$$

$$I_t^m = \frac{1}{\phi_m}\left(\frac{Q_t^m}{\Lambda_t} - 1\right) K_{t-1}^m \tag{29}$$

The above equations (26) and (27) show that the solutions for $Q_t^x$ and $Q_t^m$, which determine investment and the evolution of capital in each sector, come from forward-looking stochastic Euler equations. The shadow price or replacement value of capital in each sector is equal to the discounted value of next period's marginal productivity, the adjustment costs due to the new capital stock, and the expected replacement value net of depreciation.

We also note that the solution for each sector's $Q$ also gives each sector's investment, $I$. Alternatively, if we know the optimal decision rule for investment for each sector, we can obtain the value $Q$ for each sector:

$$Q_t^x = \Lambda_t \left(\frac{\phi_x I_t^x}{K_{t-1}^x} + 1\right)$$

$$Q_t^m = \Lambda_t \left(\frac{\phi_m I_t^m}{K_{t-1}^m} + 1\right)$$

In the steady state, of course, the investment/capital ratio is equal to the rate of depreciation for each sector. Thus, the steady state value of $Q$ for each sector is given by the following expressions:

$$\overline{Q}_t^x = \overline{\Lambda}\left(\phi_x \delta^x + 1\right)$$
$$\overline{Q}_t^m = \overline{\Lambda}\left(\phi_m \delta^m + 1\right)$$

where $\overline{\Lambda} = U'(\overline{C})$.

The solution of the model, discussed below, involves finding decision rules for $C_t, S_t, Q_t^x$, and $Q_t^m$ so that the Euler equation errors given in equations (23) through (29) are minimized. Given that we wish to impose non-negativity constraints on $C_t, S_t, I_t^x, I_t^m$ , we specify decision rules for these variables and solve for the implied values of $Q_t^x, Q_t^m$.

#### 2.1.5 Exchange rate pass-through and stickiness

The price of export goods is determined exogenously for a small open economy ($P^{x*}$) and its price in domestic currency is $P^x = SP^{x*}$. The price of import goods is also determined exogenously for a small open economy $P^{m*}$, but, we assume that price changes are incompletely passed-through (see Campa and Goldberg (2002) for a study on exchange rate pass-through and import prices). Using the definition: $P^m = SP^{m*}$ and assuming partial adjustment, we obtain:

$$p_t^m = \omega(s_t + p_t^{m*}) + (1-\omega)p_{t-1}^m \tag{30}$$

where $\omega = 1$ indicates complete pass-through of foreign price changes.

For completeness, the index of foreign price $P_t^f$ and the index of aggregate price $P_t$ are:

$$P_t^f = (1-\alpha_x)^{\alpha_x - 1}(\alpha_x)^{-\alpha_x}(P_t^x)^{\alpha_x}(P_t^m)^{1-\alpha_x} \tag{31}$$
$$P_t = (1-\alpha_f)^{\alpha_f - 1}(\alpha_f)^{-\alpha_f}\left(P_t^f\right)^{\alpha_f}(P_t^n)^{(1-\alpha_f)} \tag{32}$$

#### 2.1.6 Macroeconomic Identities

The market clearing conditions are:

$$\begin{aligned}\left(Y_t^x - \frac{\phi_x}{2K_t^x}(I_t^x)^2\right) &= (C_t^x + X_t + I_t^x)\\ \left(Y_t^m - \frac{\phi_m}{2K_t^m}(I_t^m)^2\right) &= (C_t^m - M_t + I_t^m)\\ Y_t^n &= C_t^n + G_t\end{aligned} \tag{33}$$

Real gross domestic product is given as:

$$y = \frac{1}{P_t}\left[P_t^x\left(Y_t^x - \frac{\phi_x}{2K_t^x}(I_t^x)^2\right) + P_t^m\left(Y_t^m - \frac{\phi_m}{2K_t^m}(I_t^m)^2\right) + P_t^n Y_t^n\right] \tag{34}$$

## 2.2 Terms of Trade

The only shocks explored in this paper comes from the terms of trade. Specifically:

$$p_t^{x*} = 0.9p_{t-1}^{x*} + 0.1\overline{p^{x*}} + \varepsilon_t^{x*}; \qquad \varepsilon_t^{x*} \sim N(0, 0.01)$$

where lower case denotes the log of the world export price, $p_t^{x*}$ and $\overline{p^{x*}}$ is normalized to zero. The evolution of the prices mimic actual data generating processes, with a normally distributed innovation with standard deviation set at 0.01. We assume that $p_t^{m*}$ is constant, with normalization $\overline{p^{m*}} = 0$, so that the stochastic process describes a mean-reverting terms of trade process.

The simulations are also conducted assuming that the domestic price of export goods fully reflect the exogenously determined prices:

$$p_t^x = s_t + p_t^{x*} \tag{35}$$

however, the domestic price of import goods are partially passed on:

$$p_t^m = \omega(s_t + p_t^{m*}) + (1-\omega)p_{t-1}^m \tag{36}$$

where $\omega$ is the coefficient of exchange rate pass-through and $p_{t-1}^m$ is the starting value for import goods which is set to $\overline{p^{m*}}$. In this paper we shall only present results for the case of low pass-through $\omega = 0.3$ (see estimates cited in Campa and Goldberg (2002)). This is a simulation study about the design of monetary policy for an economy subjected to relative price shocks.

## 2.3 Monetary Authority

We are concerned with Taylor (1993, 1999) type rules, one with only annualized price inflation targeting, for the desired interest rate, $\overline{i_t}$, and one with inflation and Q-growth targeting. However, they are evaluated under four scenarios. The first is the standard Taylor rule with no learning, denoted by the functions $N(\pi), N(\pi, \eta)$, the second is an optimal Tayor rule (with no learning), denoted by the functions $O(\pi), O(\pi, \eta)$.Under learning, we first use the standard Taylor rule, denoted by the functions $T(\widehat{\pi})$, $T(\widehat{\pi}, \widehat{\eta})$,and a state-contingent Taylor rule, denoted by $S(\widehat{\pi}), S(\widehat{\pi}, \widehat{\eta})$.

**Policy with No Learning**

- Standard Taylor Rules

For the pure inflation targeting regime, the desired interest rate has the following form:

$$\overline{i_t} = i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi}), \qquad \phi_\pi > 1 \tag{37}$$

with $\pi_t = ((P_t/P_{t-4}) - 1)$ representing an annualized rate of actual inflation, and $\widehat{\pi}_t$ the forecast of inflation based on central bank learning. The desired long run inflation rate is given by $\widetilde{\pi}$. The actual interest rate follows the following partial adjustment mechanism to allow for smoothing behavior:

$$i_t = \theta i_{t-1} + (1-\theta)\overline{i_t} \tag{38}$$

This no-learning Taylor rule $N(\pi)$ adopted is:

$$N(\pi) : i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\pi(\pi_t - \widetilde{\pi})\right] \tag{39}$$

In the goods price and asset price inflation regime, we change the formulation for the desired interest rate, to include the forecast of Q-growth, $\widehat{\eta}_t$ and a desired target rate, $\widetilde{\eta}$ :

$$\overline{i_t} = i^* + \phi_\pi(\pi_t - \widetilde{\pi}) + \ \phi_\eta(\eta_t - \widetilde{\eta}), \qquad \phi_\pi > 1, \phi_\eta > 0 \tag{40}$$

with $\eta_t = ((Q_t^x/Q_{t-4}^x) - 1)$ representing an annualized rate of Q-growth for exportable goods and $\widetilde{\eta}$ represents the target for this rate of growth. In this case, the Taylor rule $N(\pi, \eta)$ with smoothing becomes:

$$N(\pi,\eta) : i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\pi(\pi_t - \widetilde{\pi}) + \phi_\eta(\eta_t - \widetilde{\eta})\right] \tag{41}$$

The performance of this rule under no learning is the benchmark against which we evalute the performance of other rules.

- Optimal Taylor Rules

As a check, on the robustness of the results, we also consider the case where the Taylor coefficients are optimally determined. The rules $O(\pi)$ and $O(\pi, \eta)$ in this case are:

$$O(\pi) \ : \ i_t = \widehat{\theta} i_{t-1} + (1-\widehat{\theta})\left[i^* + \widehat{\phi}_\pi(\pi_t - \widetilde{\pi})\right] \tag{42}$$

$$O(\pi,\eta) \ : \ i_t = \widehat{\theta} i_{t-1} + (1-\widehat{\theta})\left[i^* + \widehat{\phi}_\pi(\pi_t - \widetilde{\pi}) + \widehat{\phi}_\eta(\eta_t - \widetilde{\eta})\right] \tag{43}$$

where the $\widehat{}$ indicates that they are estimated. The estimated optimal coefficients are:

$$O(\pi) \ : \ \widehat{\theta} = 0.2336; \quad \widehat{\phi}_\pi = 1.9999$$
$$O(\pi) \ : \ \widehat{\theta} = 0.000; \quad \widehat{\phi}_\pi = 1.5935, \quad \widehat{\phi}_\eta = 0.5774$$

indicating that the main implication of the fixed coefficient cases is the imposition of the smoothing coefficient.

**Policy with Learning** We now assume, perhaps more realistically, that the monetary authority does not know the exact nature of the private sector model. Instead, it adopts a VAR forecasting model of lag order $k$ for forecasting the evolution of the state variable, $x_t$. The model takes the general form:

$$x_t = \sum_{j=0}^{k} \Gamma_{1t,j} x_{t-j-1} + \Gamma_{2t} i_t + e_t$$

Under the inflation only policy scenario, the monetary authority estimates or learns the evolution of inflation as a function of its own lag as well as the interest rate. In this case $x_t = \widehat{\pi}_t$. We use six lags and $\Gamma_{1t,j}$ is a recursively updated matrix of coefficients, representing the effects of lagged inflation on current inflation.

- Linear Taylor Rule

Under learning we first use the linear framework, carried over from the case of no-learning, to this learning environment. The two Taylor rules are given by the following equations:

- $$T(\widehat{\pi}) : i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi})\right] \quad (44)$$

$$T(\widehat{\pi}, \widehat{\eta}) : i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi}) + \phi_\eta(\widehat{\eta}_t - \widetilde{\eta})\right] \quad (45)$$

- State-Contingent Taylor Rules

We also consider the case when the Taylor rule applied is dependent on the conditions at time $t$ and reflects the Central Bank's concerns about inflation. Under the inflation targeting only case, the rules $S(\widehat{\pi})$ are described in Table I.

| Table I: Policy Rules for Inflation Targeting Only: $S(\pi)$ | |
|---|---|
| | |
| $\lvert\widehat{\pi}_t\rvert < 0.02$ | $i_t = \theta i_{t-1} + (1-\theta)[i^*]$ |
| $\lvert\widehat{\pi}_t\rvert \geqq 0.02$ | $i_t = \theta i_{t-1} + (1-\theta)[i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi})]$ |

In this pure anti-inflation scenario, if the absolute value of inflation is below the target level $\pi^*$ then the government only engages in smoothing behaviour. However, if inflation is above or below the target rate, the monetary authority implements the Taylor rule.

In the inflation and Q-growth case, the state contingent Taylor policy rules $S(\widehat{\pi}, \widehat{\eta})$ are summarized in Table II.

| Table II: Policy Rules for Inflation and Q-Growth Targeting: $S(\pi, \eta)$ | | |
|---|---|---|
| | Q-Growth | |
| Inflation | $\lvert\widehat{\eta}_t\rvert < 0.02$ | $\lvert\widehat{\eta}_t\rvert \geqq 0.02$ |
| $\lvert\widehat{\pi}_t\rvert < 0.02$ | $i_t = \theta i_{t-1} + (1-\theta)[i^*]$ | $i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\eta(\widehat{\eta}_t - \widetilde{\eta})\right]$ |
| $\lvert\widehat{\pi}_t\rvert \geqq 0.02$ | $i_t = \theta i_{t-1} + (1-\theta)\left[i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi})\right]$ | $i_t = \theta i_{t-1} + (1-\theta)\left[\begin{array}{c} i^* + \phi_\pi(\widehat{\pi}_t - \widetilde{\pi}) \\ + \phi_\eta(\widehat{\eta}_t - \widetilde{\eta}) \end{array}\right]$ |

In this setup the central banks shows a strong anti-inflation or anti-deflation bias, in both the CPI and in the share price index. In other words, the central bank worries a lot about absolute inflation being above targets, either in the CPI, or in Q, or both. But it worries little about inflation or deflation in either of these variables if the absolute value of the rate is below targets. There are thus four sets of outcomes: (1) if both inflation and asset-price growth are below the target levels, then the government follows a "do no harm" cautionary approach with $\phi_\pi = \phi_\eta = 0$; (2) if inflation is above the target rate, and asset-price inflation is below the target, the monetary authority puts strong weight on CPI inflation and sets $\phi_\eta = 0$; (3) if only asset-price growth is above its target, the central bank puts strong weight on the asset-price growth target and sets $\phi_\pi = 0$; (4) if both asset-price growth and inflation are above targets, it adopts a Taylor rule on both inflation and Q-growth.

Note that monetary policy in all instances operates symmetrically. The same weight applies to inflation or growth, with different signs, when they are above or below their targets. For simplicity, with no long run inflation nor trends in terms of trade, we set the targets for inflation and growth to be zero; $\widetilde{\pi} = \widetilde{\eta} = 0$.

Uncertainly about the underlying longer-term inflation in the CPI or asset price is a rationale for our approach. Swanson (2004), for example, poses the issue as a signal extraction problem for a policy-maker, with diffuse-middle priors. In our framework, policymakers are uncertain about the underlying rate of inflation or deflation in the range [-2, 2] percent, so they are unwilling to react within this interval. As observed inflation or deflation moves further away from their prior, they react once it hits the upper/lower bounds. The main feature of this type of behaviour is "policy attentuation for small surprises" followed by "increasingly aggressive responses" at the margin [Swanson (2004): p.7].

In all scenarios, the Taylor coefficients are predetermined at $\theta = 0.9, \phi_\pi = 1.5$ and $\phi_\eta = 0.5$.

# 3 Calibration and Solution Algorithm

In this section we discuss the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model. We then summarize the parameterized expectations algorithm (PEA) used for solving the model.

## 3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table III.

| Table III: Calibrated Parameters | |
|---|---|
| Consumption | $\gamma = 3.0$ , $\beta = 0.009$ |
| | $\alpha_x = 0.5$, $\alpha_f = 0.5$ |
| Production | $\theta_m = 0.7$, $\theta_x = 0.3$ |
| | $\delta_x = \delta_m = 0.025$ |
| | $\phi_x = \phi_m = 0.03$ |

Many of the parameter selections follow Mendoza (1995). The constant relative risk aversion $\gamma$ is set at 3.0 (to allow for high interest sensitivity). The shares of non-traded goods in overall consumption is set at 0.5, while the shares of exports and imports in traded goods consumption is 50 percent each. Production in the export goods sector is more capital intensive than in the import goods sector.

The initial values of the nominal exchange rate, the price of non-tradeables and the price of importable and exportable goods are normalized at unity while the initial values for the stock of capital and financial assets (domestic and foreign debt) are selected so that they are compatible with the implied steady state value of consumption, $\overline{C} = 2.02$, which is given by the interest rate and the endogenous discount factor. The values of $\overline{C}^x$, $\overline{C}^m$, and $\overline{C}^n$ were calculated on the basis of the preference parameters in the sub-utility functions and the initial values of $B$ and $L^*$ deduced. The steady-state level of investment for each sector is equal to the depreciation rate multiplied by the respective steady-state capital stock.

Similarly, the initial shadow price of capital for each sector is set at its steady state value. The production function coefficients $A^m$ and $A^x$, along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector. Since the focus of the study is on the effects of terms of trade shocks, the domestic productivity coefficients were fixed for all the simulations.

Finally, the foreign interest rate $i^*$ is also fixed at the annual rate of 0.04. In the simulations, the effect of initialization is mitigated by discarding the first 100 simulated values.

## 3.2 Solution Algorithm and Constraints

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), we parameterize nonlinear decision rules for $C_{t},_{t}$ $S_t$, $I_t^x, I_t^x$, given by $\psi^S, \psi^C, \psi^{I^x}$, and $\psi^{I^f}$:

$$
\begin{aligned}
C_t &= \psi^C(\mathbf{x}_{t-1}; \Omega_C) && (46)\\
S_t &= \psi^S(\mathbf{x}_{t-1}; \Omega_S) && (47)\\
I_t^x &= \psi^{I^x}(\mathbf{x}_{t-1}; \Omega_{Q^x}) && (48)\\
I_t^m &= \psi^{I^m}(\mathbf{x}_{t-1}; \Omega_{Q^m}) && (49)
\end{aligned}
$$

The parameters of these decision rules are selected to minimize the squared Euler-equatio errors given in (23) to (29):

The symbol $\mathbf{x}_{t-1}$ represents a vector of observable state variables known at time $t$: the terms of trade, the capital stock for exports and manufacturing goods, the level of foreign debt and the interest rate, relative to their steady state values:

$$
\mathbf{x}_t = \ln\left[\frac{P_t^{x*}}{P_t^{m*}}, \frac{K_{t-1}^x}{\overline{K}^x}, \frac{K_{t-1}^m}{\overline{K}^m}, \frac{L_{t-1}^*}{\overline{L}^*}, \frac{1+i_{t-1}}{1+\overline{i}}\right] \qquad (50)
$$

The symbols $\Omega_\lambda, \Omega_S, \Omega_{Q^x}$, and $\Omega_{Q^m}$ represent the parameters for the expectation function, while $\psi^C, \psi^E, \psi^{Q^x}$ and $\psi^{Q^f}$ are the expectation approximation functions.[7]

Judd (1996) classifies this approach as a "projection" or a "weighted residual" method for solving functional equations, and notes that the approach

[7] In the case of no learning and optimal Taylor rules, the coefficients in the Taylor rules are jointly estimated with the parameters of the expectation functions.

was originally developed by Williams and Wright (1982, 1984). The functional forms for $\psi^E, \psi^C$, $\psi^{Q^x}$, and $\psi^{Q^f}$ are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks can produce results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

We use a neural network specification with two neurons for each of the decision variables. The neurons take on values between [0, 1] for a logsigmoid function and between [-1, 1] for a tansigmoid function The functions were then weighted by coefficients, and an exponent or anti-log function applied to the final value. The functions were multiplied by the steady state values to ensure steady state convergence.

The model was simulated for repeated parameter values for $\{\Omega_C, \Omega_S, \Omega_{Q^x}, \Omega_{Q^m}\}$ and convergence obtained when the expectation errors were minimized. In the algorithm, the following non-negativity constraints for consumption and the stocks of capital were imposed by the functional forms of the approximating functions:

$$C_t^x > 0, \quad K_t^x > 0, \quad K_t^m > 0, \quad I_t^x > 0, \quad I_t^m > 0, \quad i_t > 0 \tag{51}$$

The usual no-Ponzi game applies to the evolution of real government debt and foreign assets, namely:

$$\lim_{t\to\infty} B_t \exp^{-it} = 0, \qquad \lim_{t\to\infty} L_t^* \exp^{-(i^*+\Delta s_{t+1})t} = 0 \tag{52}$$

We keep the foreign asset or foreign debt to GDP ratio bounded, and thus fulfill the transversality condition, by imposing the following constraint on the parameterized expectations algorithm:[8]

$$\sum \left( \frac{|S_t L_t^*| / P_t}{y_t} \right) < \widetilde{L}, \qquad \sum \left( \frac{|B_t| / P_t}{y_t} \right) < \widetilde{B} \tag{53}$$

where $\widetilde{L}$, and $\widetilde{B}$ are the critical foreign and domestic debt ratios.

# 4 Simulation Analysis

## 4.1 Base-Line Results

The aim of the simulations is to compare the outcome for consumption, inflation and welfare for the two policy scenarios - inflation targeting ($\pi$)

[8]In the PEA algorithm, the error function will be penalized if the foreign debt/gdp ratio is violated. Thus, the coefficients for the optimal decision rules will yield debt/gdp ratios which are well below levels at which the constaint becomes binding.

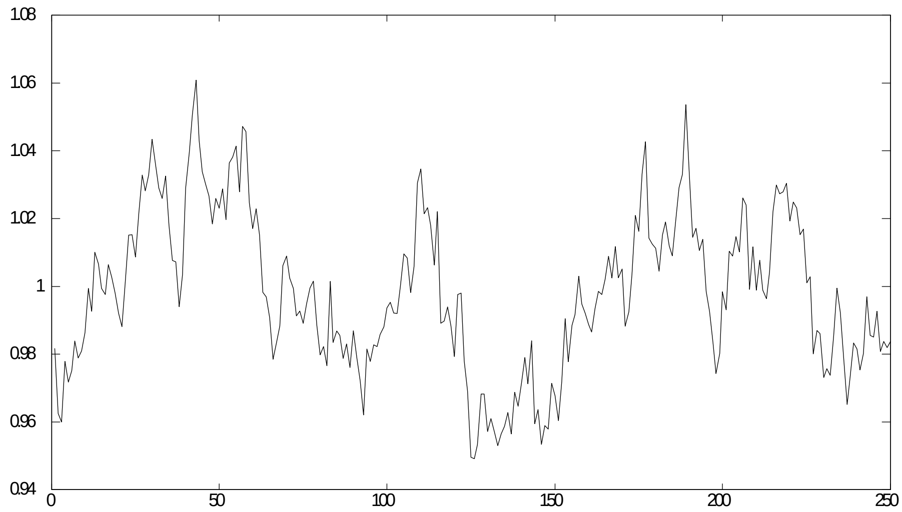


Figure 1: One realization of the terms of trade shocks

and inflation and $Q$-growth targeting ($\pi$ and $\eta$). To ensure that the results are robust, we conducted 1000 simulations (each containing a time-series of 250 realizations of terms of trade shocks) for the case of relatively low pass-through ($\omega = .3$).

Figure 1 shows the simulated paths for one time series realization of the exogenous terms of trade index. Figure 2 pictures the paths with learning, with the solid lines are for the case of pure inflation targets, while the dotted lines are for the case of inflation and Q-growth targeting. The simulated values for other variables (not shown) are also well-behaved.

In the learning scenarios, we note that, despite the large swings in the terms of trade index, consumption is more stable with the inclusion of Q-growth targeting under both Taylor frameworks. We also see that both inflation and Q-growth do fall outside the bounds of [-0.02 0.02] for the state-contingent framework. However, the violations of these bounds are not persistent, neither in the case of pure CPI nor in the case of CPI and asset-price inflation targets.

To ascertain which policy regime yields the higher welfare value, we examined the distribution of the welfare outcomes of the different policy regimes for 1000 different realizations of the terms of trade shocks. Before presenting these results, we present the accuracy checks of the simulation results as well as the "rationality" of the learning mechanism.

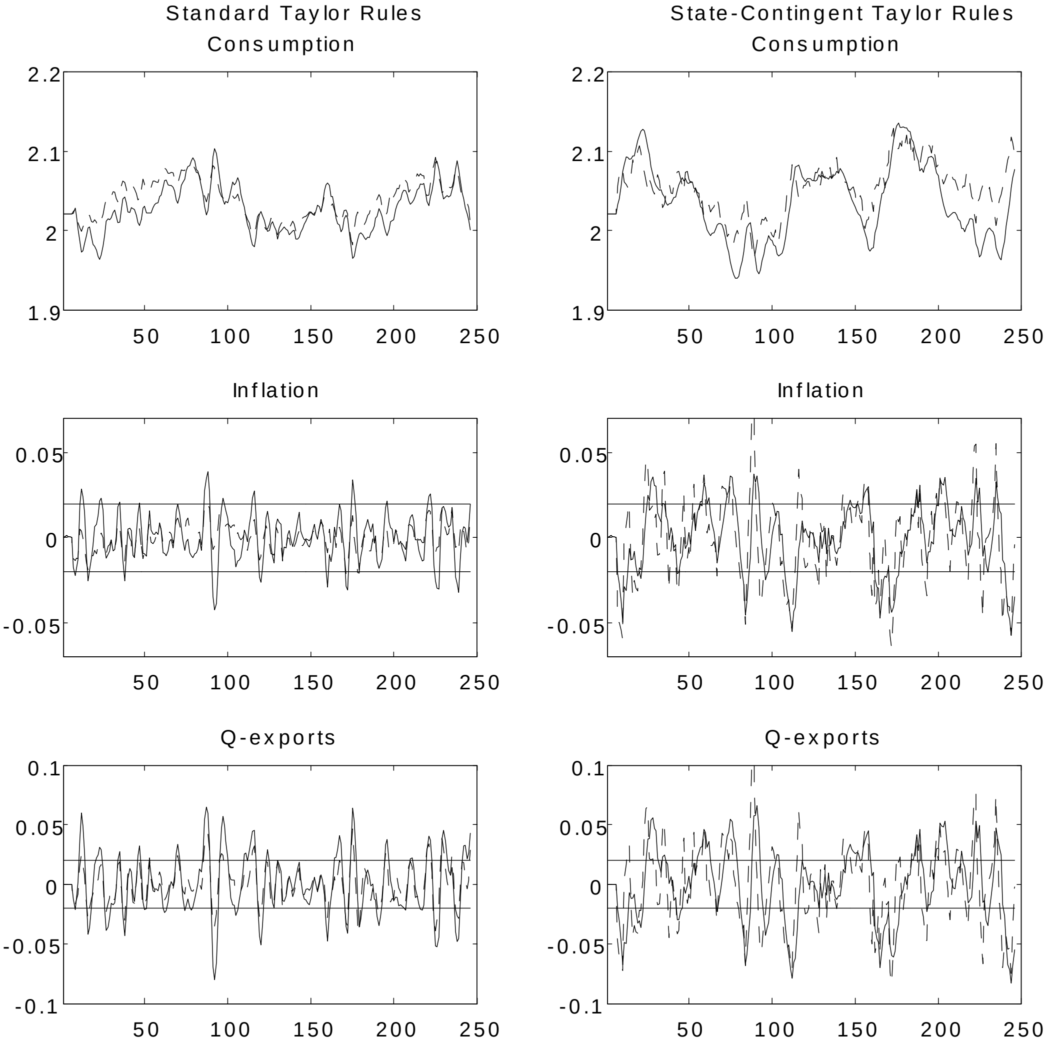


Figure 2: Time Series under Standard/State-Contingent Taylor Rules _pure inflation targets _ _ CPI/asset price inflation targets

## 4.2 Accuracy Test

The accuracy of the simulations may be checked by the Judd-Gaspar statistic which is the maximum value of the absolute value of the Euler equation error for consumption $\nu_t$ relative to $C_t$. That is, for realization $j$, with size $T$, the accuracy measure is:

$$\begin{aligned} JG^{(j)}_{\max} &= \max\left[\frac{\mid \nu_t \mid}{C_t}\right] \\ \text{where} \quad \nu_t &= \vartheta_{t+1}\Lambda_{t+1}\frac{1}{P_{t+1}}(1+i_t) - \vartheta_t\Lambda_t\frac{1}{P_t} \end{aligned}$$

This statistic is a measure of the maximum error relative to a dollar spent on consumption for each realization.

Table IV present the means and standard deviations of the Judd-Gaspar accuracy measures based on the maximum absolute error measures. We see that the average size of the accuracy error measures are in the range 0.16 to 0.32 of one cent for every dollar spent on consumption.

Table IV: Judd-Gaspar Accuracy Statistic: Maximum Absolute Error Mean and Standard Deviation (in parenthesis)

| | |
|---|---|
| **Taylor Rule Framework with Learning** | |
| Inflation Targeting: $T(\widehat{\pi})$ | 0.0024 (0.0004) |
| Inflation/Q-Growth Targeting: $T(\widehat{\pi}, \widehat{\eta})$ | 0.0016 (0.0004) |
| **State Contingent Taylor Rule with Learning** | |
| Inflation Targeting: $S(\widehat{\pi})$ | 0.0017 (0.0003) |
| Inflation/Q-Growth Targeting: $S(\widehat{\pi}, \widehat{\eta})$ | 0.0032 (0.0006) |
| **Taylor Rule with Fixed Coefficients and No Learning** | |
| Inflation Targeting: $N(\pi)$ | 0.0016 (0.0003) |
| Inflation/Q-Growth Targeting: $N(\pi, \eta)$ | 0.0023 (0.0004) |
| **Taylor Rule with Optimal Coefficients and No Learning** | |
| Inflation Targeting: $O(\pi)$ | 0.0016 (0.0003) |
| Inflation/Q-Growth Targeting: $O(\pi, \eta)$ | 0.0018 (0.0003) |

## 4.3 Learning and Quasi-Rationality

In our model, the central bank learns the underlying process for inflation in the pure inflation-target regime and the underlying processes for inflation and growth in the inflation-Q-growth targeting regime. Learning takes the form of recursive updating of the least-squares estimates of a vector autoregressive model.

Marcet and Nicolini (1997) raise the issue of reasonable rationality requirements in their discussion of recurrent hyperinflation and learning behavior. In our model, a similar issue arises. Given that the only shocks in the model are recurring terms of trade shocks, with no abrupt, unexpected structural changes taking place, the learning behavior of the central bank should not depart, for too long, from the rational expectations paths. The central bank, after a certain period of time, should develop forecasts which converge to the true inflation and growth paths of the economy, unless we wish to make some special assumption about monetary authority behavior.

Marcet and Nicolini discuss the concepts of "asymptotic rationality", "epsilon-delta rationality" and "internal consistency", as criteria for "boundedly rational" solutions. They draw attention to the work of Bray and Savin (1986). These authors examine whether the learning model rejects serially uncorrelated prediction errors between the learning model and the rational expectations solution, with the use of the Durbin-Watson statistic. Following Bray and Savin, we also use the Durbin-Watson statistic to examine whether the learning behavior is "boundedly rational".

Table V presents the Durbin-Watson statistics for the inflation and Q-growth forecast errors of the central bank, under both policy regimes. In all of the cases, we see that the learning behavior does not violate near rationality. This suggests that differences between the learning and no-learning case should be small. Consequently, we shall focus on the learning results in the subsequent sections.

Table V: Durbin-Watson Statistics for Forecast Errors
Percentage in Lower and Upper Critical Regions

| | Inflation | Q-Growth |
|---|---|---|
| Standard Tayor Rule | | |
| Inflation Targeting: $T(\widehat{\pi})$ | 0/0 | — |
| Inflation/ Q-Growth Targeting: $T(\widehat{\pi}, \widehat{\eta})$ | 0/0 | 0/0 |
| State-Contingent Taylor Rules | | |
| Inflation Targeting: $S(\widehat{\pi})$ | 0/0 | — |
| Inflation/ Q-Growth Targeting: $S(\widehat{\pi}, \widehat{\eta})$ | 0/0 | 0/0 |

## 4.4 Comparative Results

This section summarizes the results for 1000 alternate realizations of the terms-of-trade shocks (each realization contains 250 observations), for the Taylor rule and the State-Contingent Taylor rules for conducting monetary policy with learning. Table VI presents the mean and standard deviation of the coefficients of variations of the 1000 samples for consumption, inflation and Q-exports.

Table VI: Summary Statistics of the Coefficient of Variation
Mean (Standard Deviations in Parentheses)

| | $T(\widehat{\pi})$ | $T(\widehat{\pi},\widehat{\eta})$ |
|---|---|---|
| Consumption | 0.0119 (0.0016) | 0.0098 (0.0022) |
| Inflation | 0.0138 (0.0013) | 0.0071 (0.0011) |
| Q-growth | 0.0251 (0.0025) | 0.0155 (0.0029) |
| | $S(\widehat{\pi})$ | $S(\widehat{\pi},\widehat{\eta})$ |
| Consumption | 0.0199 (0.0033) | 0.0150 (0.0021) |
| Inflation | 0.0223 (0.0026) | 0.0254 (0.0022) |
| Q-growth | 0.0343 (0.0039) | 0.0347 (0.0030) |

The simulation results across policy frameworks (with pre-set Taylor rule coefficients) show a fall in the coefficient of variation for consumption, when we change from an inflation only to an inflation/Q-growth regime. As for the target variables, the coefficient of variations for inflation and Q-growth fell under standard Taylor rules, but they increased under the state-contingent scheme.

## 4.5 Welfare Implications

Figure 4 shows the welfare differences for different comparisons of the 4 possible regimes, with learning, considered in the paper. These distributions show that $S(\widehat{\pi},\widehat{\eta})$ unambiguously generates better welfare outcomes compared to $S(\widehat{\pi})$ and $T(\widehat{\pi},\widehat{\eta})$, and, on average for the majority of times, generates better welfare outcomes than the simpliest framework $T(\widehat{\pi})$.

Following Schmitt-Grohe and Uribe (2004), we also computed the average consumption compensation necessary for a household to be as well off in the reference regime compared to the alternative. Using the relationship below

$$U((1-\lambda)C_t^r) = U(C_t^a)$$

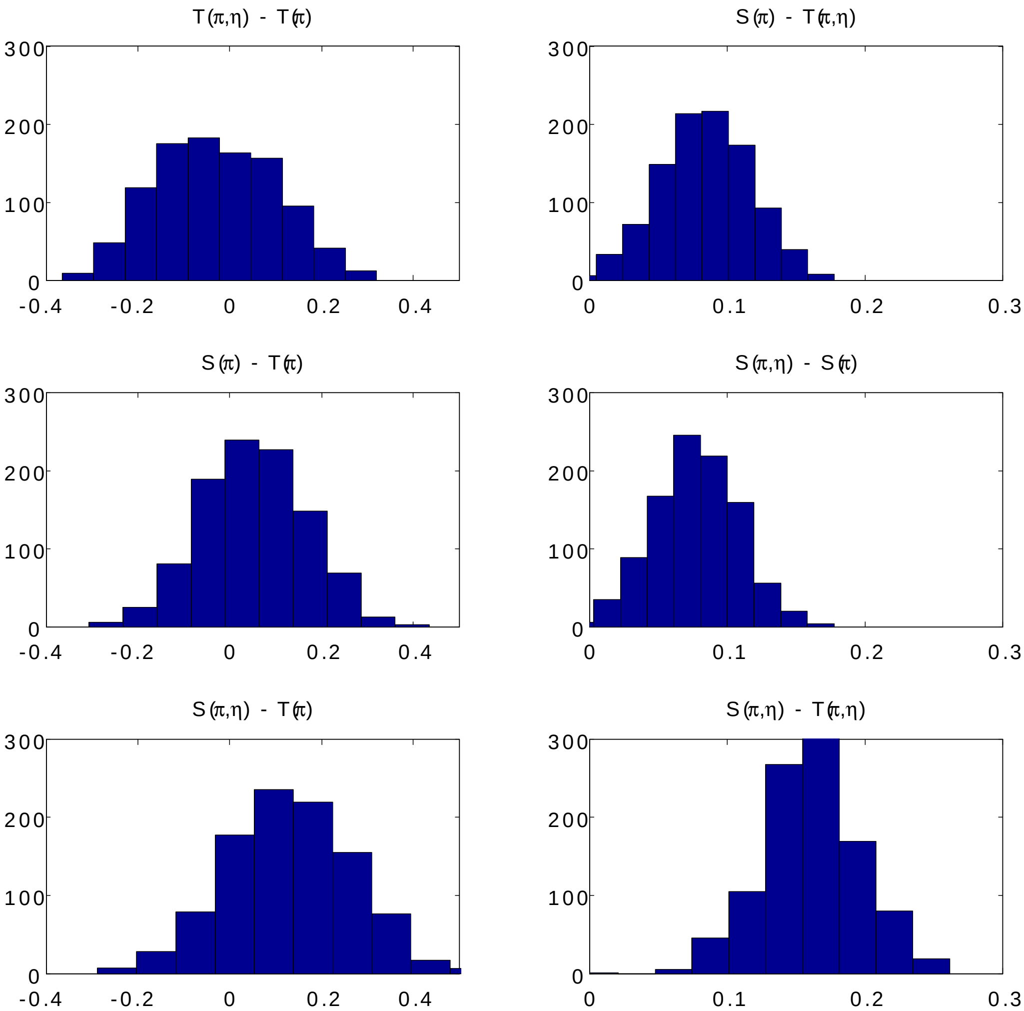


Figure 3: Welfare Differences Under Rules with Learning

and the utility function and the welfare functions in (1) and (2) respectively yields:

$$\lambda\% = \left[ 1 - \left( \frac{W^a}{W^r} \right)^{\frac{1}{(1-\gamma)}} \right] \times 100$$

Positive values indicates what households can give up and be as well off under the alternative regime compared to the reference regime, negative values indicate the consumption compensation necessary for households to be as well off. As shown in Table VII, a household could give up 0.1295% of the consumption in a regime with standard Taylor rules and inflation only targeting $T(\widehat{\pi})$ and be as well off in a policy framework with standard Taylor rules with inflation and Q-growth targeting $T(\widehat{\widehat{\pi}}, \widehat{\eta})$. In other words, it is, on average, welfare-reducing. In contrast, a household would need to be compensated by 0.2409% of the consumption in a $T(\widehat{\pi})$ policy regime to be as well off in a policy framework with state-contingent Taylor rules with inflation targeting $S(\widehat{\pi})$. In other words, it is, on average, welfare-improving. Overall, these results show that a household can be better off, in a learning regime with state-contingent Taylor rules. The best improvements come from switching from a pure inflation targeting regime with either simple or state-contingent Taylor rules ($T(\widehat{\pi})$ or $S(\widehat{\pi})$) to the state-contingent Taylor rule aimed at both inflation and Q-growth $S(\widehat{\pi}, \widehat{\eta})$. The consumption gain is 0.59 and 0.72 per cent respectively.

| Table VII: Consumption Compensation (%) | | |
|---|---|---|
| Policy Frameworks | | |
| reference | alternative | |
| $T(\widehat{\pi})$ | $T(\widehat{\pi}, \widehat{\eta})$ | 0.1295 |
| $T(\widehat{\pi})$ | $S(\widehat{\pi})$ | -0.2409 |
| $T(\widehat{\pi})$ | $S(\widehat{\pi}, \widehat{\eta})$ | -0.5900 |
| $T(\widehat{\pi}, \widehat{\eta})$ | $S(\widehat{\pi})$ | -0.3714 |
| $T(\widehat{\pi}, \widehat{\eta})$ | $S(\widehat{\pi}, \widehat{\eta})$ | -0.3477 |
| $S(\widehat{\pi})$ | $S(\widehat{\pi}, \widehat{\eta})$ | -0.7203 |

# 5 Concluding Remarks

This paper has been concerned with incorporating the rate of growth of Tobin's Q as an additional target to inflation for monetary policy in a learning environment. Our results show that the Central Bank improves welfare if it targets asset-price as well as consumer price inflation. However, the best way to introduce Q-growth is not with a linear Taylor rule, but with a state-contingent Taylor-rule framework. To be sure, we did not introduce shocks in this model in the form of asset price bubbles.[9]

Under no learning, or model certainty, the advantages of targeting Q-growth in addition to CPI inflation disappears. The economic intuition behind our result, that asset-price inflation helps only in a learning context, is that including Q in the formulation of monetary policy improves the central bank's forecasting of inflation. Since the central bank does not know the true model, including asset-price growth brings into the policy process more foreward-looking information and improves the effectiveness of monetary policy. When the forecast of inflation is based on the true model, of course, there is no need to make use of this added foreward-looking information. .

We assumed that the driving force for Q growth comes from fundamentals, both in the underlying model and in the learning process. Given that the Central bank has to learn the laws of motion of Q-growth as well as inflation, and set policy on the basis of longer-term laws of motion of these variables, it seems reasonable to start with Q driven solely by fundamentals. We leave to further research an examination of the robustness of our results to the incorporation of bubbles and other non-fundamental asset-price shocks.

Finally, we note that our time-varying state-contingent interest-rate rules, coming from uncertainty about the true laws of motion of consumer and asset-price inflation dynamics, generated by a nonlinear stochastic model, is a step away from the design of a nonlinear interest-rate rule, in which the laws of motion are approximated by nonlinear approximation methods. It may be that nonlinear policy rules may show even more beneficial effects from a cautionary monetary policy aimed at asset price as well as consumer price inflation.

[9]See Dupor (2005) for a closed economy study of whether monetary policy should respond to asset price fluctuations which are not driven by fundamentals.

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