Do Tax Cuts Matter?
Supply-Side Results from a New Keynesian Model

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Abstract

This paper shows that temporarily cutting tax rates on income or consumption are effective mechanisms for reducing government debt, both when steady-state interest rates are at the zero lower bound, when expansionary monetary policy is not an option, and when interest rates are high, when inflationary monetary policy can be used.

JEL Classificaltion: E52, E62,F41

1 Introduction

The paper examines alternatives to monetary policy as a debt stabilisation tool when interest rates are very close to the zero lower bound. Even when interest rates are not at the zero bound, inflating the debt away is not a serious option. Under central bank independence, the mandate for monetary policy is inflation control and thereby dampen the business cycle, not reduce the interest expenses of the fiscal authority by inflating away debt. This means that the reduction in debt needs to come from current or future increases in tax revenues.

Romer and Romer (2010) found that tax rate changes were particularly effective instruments for stimulating output, and thus governmnet tax revenue, if the particular changes were set, for example, to reduce debt. However they are silent about whether tax rate effects are working through labor supply behavior or through consumption demand stimulus. Correia, Fahri, Nicolini and Teles (2010) point out, once monetary and fiscal policies are jointly considered, the zero lower bound on interest rates is not a constraint on policy even during a severe recession [Correa, Fahri, Nicolini and Teles (2010): p. 3]. Fiscal tools such as tax rates on income and consumption may be used to stimulate current spending and generate tax revenue for reducing debt.

The aim of the paper is to compare the effectiveness of government debt-continent rules for interest rates as well as for tax rates (on labor income and consumption) as a means to reduce the size of the debt. Our experiment is quite different from the Clinton, Kumhof, Laxton, and Mursula (2011) deficit-reduction simulation with the global IMF model. In our experiment, the debt, while large, is starting to fall, but it does not fall fast enough. This issue for us is how to reduce public debt (which came about because of a large initial expansion in government spending), but when government spending consolidation is already underway. We consider the case when interest rates are close to the zero lower bound, and when they are not.

To anticipate the results, we find that state-contingent tax cuts, like reduced interest rates, increase consumption and tax revuenue, and reduce government debt. The tax rate cuts are more powerful that interest rate cuts and of course operate when the interest rates are near the zero lower bound.

Section 2 present theclosed economy model. To ensure that the model results bear some semblance to reality, we present some estimates of the key parameters in section 3. The next section takes up the nonlinear Parameterized Expectation Algorithm (PEA) for solving the model. Then we apply robustness checks to our results.

2 The Model

The model contains a detailed household sector and detailed policy rules about the budget deficit.

2.1 Households and Calvo Wage Setting Behavior

A household typically chooses the paths of consumption, labor, money and bonds to maximize the present value of its utility function $U(C, L_t)$ subject to the budget constraint: The objective function of the household is given by the following expression:

$$
\begin{aligned}
\underset{\{C_t,L_t,B_t\}}{Max}\ \mathcal{L} &= \sum_{\iota=0}^{\infty}\beta^{\iota}\left\{\begin{array}{c} U(C_{t+\iota},L_{t+\iota}) \\ -\Lambda_{t+\iota}\left[\begin{array}{c} P_{t+\iota}C_{t+\iota}(1+\tau^{c}_{t+\iota})+B_{t+\iota} \\ -(1+R_{t-1+\iota})B_{t-1+\iota} \\ -(1-\tau^{w}_{t+\iota})W_{t+\iota}L_{t+\iota}-\Pi_{t+i} \end{array}\right] \end{array}\right\} \quad (1) \\
U(C,L_t) &= \frac{(C_t)^{1-\eta}}{1-\eta}-\frac{L_t^{1+\varpi}}{1+\varpi} \quad (2)
\end{aligned}
$$

Overall utility is a positive function of consumption and real balances, and and a negative function of labor[1]; the parameter η is the relative risk aversion coefficient, ϖ is the Frisch labor supply elasticity, while β represents the constant, exogenous discount factor. In addition to buying consumption good, C_t, households hold government bonds B_t which pays return R_t, and receive dividends from the firms Π_t. The household pays taxes on labor income $\tau^w_t W_t L_t$, on consumption expenditures $\tau^c_t P_t C_t$.

In this paper we assume that wages are set as staggered contracts. A fraction $(1-\xi)$ of households renegotiate their contracts each period. Each household chooses the optimal wage W^o_t by maximizing the expected discounted utility subject to the demand for its labor L^h_t :

$$L^h_t = \left(\frac{W^o_t}{W_t}\right)^{-\zeta} L_t$$

Taking derivative with respect to W^o_t yields the first order condition:

$$E_t\sum_{\iota=0}^{\infty}\left\{\begin{array}{c} \xi^{\iota}\beta^{\iota}\left(-L^{\varpi}_{t+\iota}\right)\left(W_{t+\iota}\right)^{\zeta}L_{t+\iota}\left[-\zeta\left(W^o_{t+\iota}\right)^{-\zeta-1}\right] \\ +\Lambda_{t+\iota}(1-\tau^w_t)\left(W_{t+\iota}\right)^{\zeta}L_{t+\iota}\left[(-\zeta+1)\left(W^o_{t+\iota}\right)^{-\zeta}\right] \end{array}\right\}=0$$

which in turn can be rearranged as (assuming the usual assumption of a subsidy to eliminate the mark-up effects:

$$\left(W^a_t\right)^{1+\zeta\varpi} = \frac{\sum_{t=0}^{\infty}\xi^t\beta^t\left(W_t\right)^{\zeta+\zeta\varpi}\left(L_t^{1+\varpi}\right)}{\sum_{t=0}^{\infty}\xi^t\beta^t\Lambda_t(1-\tau^w_t)\left(W_t\right)^{\zeta}L_t}$$

Note that, in the steady-state (or when $\xi=0$), this collapse to the same condition as the competitive case:

$$
\begin{aligned}
(W)^{1+\zeta\varpi} &= \frac{(W)^{\zeta\varpi}\left(L^{\varpi}\right)}{\Lambda(1-\tau^w)} \\
W &= \frac{\left(L^{\varpi}\right)}{\Lambda(1-\tau^w)}
\end{aligned}
$$

[1] The coefficient of the disutility of labor is set at unity.

The wage equation can be rewritten using auxiliary equations A_t^{w1} and A_t^{w2}:

$$
\begin{aligned}
A_t^{w1} &= (W_t)^{\zeta+\zeta\varpi}\left(L_t^{1+\varpi}\right) + \xi\beta.A_{t+1}^{w1} \\
A_t^{w2} &= \Lambda_t(1-\tau_t^w)\,(W_t)^{\zeta}\,L_t + \xi\beta.A_{t+1}^{w2} \\
(W_t^o)^{1+\zeta\varpi} &= \frac{A_t^{w1}}{A_t^{w2}} = \frac{(W_t)^{\zeta+\zeta\varpi}\left(L_t^{1+\varpi}\right) + \xi\beta.A_{t+1}^{w1}}{\Lambda_t(1-\tau_t^w)\,(W_t)^{\zeta}\,L_t + \xi\beta.A_{t+1}^{w2}}
\end{aligned}
$$

with aggregate wage equation as:

$$
W_t = \left[\xi\,(W_{t-1})^{1-\zeta} + (1-\xi)(W_t^o)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{3}
$$

The Euler equations implied by household optimization of its intertemporal utility with respect to C_t, L_t and B_t are:

$$
\begin{aligned}
C_t^{-\eta} &= \Lambda_t P_t(1+\tau_t^c) && (4)\\
L_t^{\varpi} &= \Lambda_t(1-\tau_t^w)W_t && (5)\\
\Lambda_t &= \beta\Lambda_{t+1}(1+R_t) && (6)
\end{aligned}
$$

The first equation, 4 tells us that the marginal utility of consumption, divided by the tax-adjusted price level, is equal to the marginal utility of wealth Λ_t. The second equation, 5, relates the marginal dis-utility of labour, adjusted by the after-tax wage, to the foregone marginal utility of consumption. The third equation is the Keynes-Ramsey rule for optimal saving: the marginal utility of wealth today should be equal to the discounted marginal utility tomorrow, multiplied by the tax-adjusted gross rate of return on saving.

2.2 Production and Calvo Price Setting Behavior

Output is a function of labor only (that is we abstract from issues associated with capital formation).

$$
\begin{aligned}
Y_t &= Z_t L_t && (7)\\
\log(Z_t) &= \rho_z \log(Z_{t-1}) + (1-\rho_z)\log(\overline{Z}) + \epsilon_{z,t}; \quad \epsilon_{z,t} \sim N(0,\sigma_z^2) && (8)
\end{aligned}
$$

where the productivity term Z_t is assumed to follow a simple exogenous autoregressive process. Total output is for both household and government consumption:

$$
\begin{aligned}
Y_t &= C_t + G_t && (9)\\
\log(G_t) &= \rho_g \log(G_{t-1}) + (1-\rho_g)\log(\overline{G}) + \epsilon_{g,t}; \quad \epsilon_{g,t} \sim N(0,\sigma_g^2) && (10)
\end{aligned}
$$

where government spending G_t is assumed to follow a simple exogenous autoregressive process, with autoregressive coefficient ρ_g,steady state $\overline{G}$, and a stochastic shock $\epsilon_{g,t}$ normally distributed with mean zero and variance σ_g^2.

The profits of the firms are given by the following relation, and distributed to the households:

$$\Pi_t = P_t Y_t - W_t L_t \tag{11}$$

We assume sticky monopolistically competitive firms. In the Calvo price setting world, there are forward-looking domestic-goods price setters and backward looking setters. Assuming at time t that ξ is the probability of persistence, with demand for the product from firm j given by $Y_t \left(P_t^j / P_t^c\right)^{-\zeta}$, the optimal domestic-goods price, P_t^o can be written in forward recursive formulation as:

$$A = W_t / Z_t \tag{12}$$

$$P_t^o = \frac{N_t}{D_t} \tag{13}$$

$$N_t = Y_t \left(P_t\right)^{\zeta} A_t + \beta \xi N_{t+1} \tag{14}$$

$$D_t = Y_t \left(P_t\right)^{\zeta} + \beta \xi D_{t+1} \tag{15}$$

$$P_t = \left[\xi \left(P_{t-1}\right)^{1-\zeta} + (1-\xi)\left(P_t^o\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{16}$$

where A_t is the marginal cost at time t, while the domestic price level P_t is a CES aggregator of forward and backward-looking prices.

2.3 Monetary and Fiscal Policy

The Central Bank is responsible for monetary policy and it is assumed to adopt an inflation-targeting Taylor rule, with smoothing, but which respects the zero lower bound on interest rates. The Taylor rule takes the following form:

$$(R_t) = \rho_r (R_{t-1}) + (1-\rho_r)(\overline{R} + \phi_\pi(\pi_t - \pi^*)); \quad \phi_\pi > 1 \tag{17}$$

where the variable π_t is the inflation rate at time t, π^* is the target inflation rate, and ρ_r is the smoothing coefficient, with $0 < \rho_r < 1$. The parameter ϕ_π is the Taylor rule inflation coefficient., and $\overline{R}$ is the steady state interest rate.

However, in the high interest rate economy, we allow the central bank to abandon (temporarily) the inflation-targeting Taylor regime in favor of a policy aimed at debt reduction:

$$R_t = \rho_r R_{t-1} + (1-\rho_r)[\overline{R} + \phi_B(B_{t-1} - B^*)]; \quad \phi_B < 0 \tag{18}$$

In this scenario, B^* is the target debt level and ϕ_B is the coefficient relating debt to interest rate changes.

The Treasury is responsible for fiscal policy and we assume that it is implemented via tax rates on labor income and consumption, which changes according to the following rules:

$$\tau_t^w = \rho_w \tau_{t-1}^w + (1-\rho_w)\tau_0^w + (1-\rho_w)\tau_w(B_{t-1} - B^*) \tag{19}$$

$$\tau_t^c = \rho_c \tau_{t-1}^c + (1-\rho_c)\tau_0^c + (1-\rho_c)\tau_c(B_{t-1} - B^*) \tag{20}$$

The steady-state non-contingent tax rates are τ_0^w and τ_0^c. The tax rates have persistence coefficients ρ_w and ρ_c, which allows for some intertia in the adjustment of the tax rates to changes in debt. The coefficients τ_w, τ_c are set to values less than one.

The final equation is the fiscal borrowing requirement given as follows:

$$B_t = (1 + R_{t-1})B_{t-1} + P_t G_t - \tau_t^w W_t L_t - \tau_t^c P_t C_t \tag{21}$$

2.4 Log-Linearized Base Model

For the log-linearized version of the model, we posit three shocks - for productivity, government spending, and to the Taylor rule:

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \epsilon_t^z \\ g_t &= \rho_g g_{t-1} + \epsilon_t^g \\ r_t &= \rho_r r_{t-1} + (1-\rho_r)\phi_\pi \pi_t + \epsilon_t^r \\ \text{for } i &= z, g, r : \epsilon_t^i \tilde{} N(0, \sigma_i^2) \end{aligned}$$

The log-linearized Euler equation for consumption, the production function with productivity, the GDP identity, the Calvo equations for price and wage inflation, and the government budget constraint are given by the following equations:

$$\begin{aligned} -\eta c_t &= -\eta c_{t+1} - (\pi_{t+1}) + r_t && (22) \\ y_t &= z_t + l_t && (23) \\ y_t &= \frac{\overline{C}}{\overline{Y}} c_t + \frac{\overline{G}}{\overline{Y}} g_t && (24) \\ \xi(\pi_t) - \beta\xi(\pi_{t+1}) &= (1-\xi)(1-\beta\xi)(w_t - p_t - z_t) && (25) \\ (1+\zeta\varpi)\xi\pi_t^w - (1+\zeta\varpi)\beta\xi\pi_{t+1}^w &= (1-\xi)(1-\beta\xi)[\varpi l_t + \eta c_t - r w_t] && (26) \\ \begin{bmatrix} \tau^w \frac{\overline{WL}}{\overline{PY}}(w_t - p_t + l_t) + \tau^c \frac{\overline{C}}{\overline{Y}} c_t \\ + \frac{\overline{B}}{\overline{PY}}(b_t - p_t) \end{bmatrix} &= \begin{bmatrix} \frac{\overline{G}}{\overline{Y}}(g_t) \\ + \frac{\overline{B}}{\overline{PY}} \overline{R}((b_{t-1} - p_{t-1}) - \pi_t + r_{t-1}) \end{bmatrix} && (27) \end{aligned}$$

Given the steady-state tax rates and the consumption, government spending, and real labor income shares, the steady-state bond/GDP ratio is implied by the government budget constraint:

$$(1 - \overline{R})\frac{\overline{B}}{\overline{PY}} = \frac{\overline{G}}{\overline{Y}} - \overline{\tau^c}\frac{\overline{C}}{\overline{Y}} - \tau^w \frac{\overline{WL}}{\overline{PY}}$$

3 Parameter Configurations for Japan and the USA

Japan has been experiencing low interest rates since the 1990's, , while interest rates in the US fell to almost zero in the last decade, and have been at that level since. For this reason, we have selected to obtain estimates of the deep parameters based on these two countries. Table 3 presents the parameter estimates for Japan and the United State with quarterly data for the period 1990-2008.1. The table presents the distributions for the parameters and shock volatilities as well as the prior mean and standard deviations, with Bayesian estimation.

Regarding our choice of priors, we follow closely the existing literature. For each parameter, we specify the distribution, the mean, as well as the standard deviation. The volatility priors have inverse gamma distributions. Parameters restricted to fall between zero and one have a beta distribution, while coefficients outside this range are specified with a normal distribution with restrictions on their infimum and supremum. The choice of prior distributions as well as their mean and standard deviation values closely match those used by Smets and Wouters (2007). Regarding the posterior distribution, we present the mode, mean and and lower and upper 5% confidence levels of the posterior distributions. The estimates come from Metropolis-Hastings Monte Carlo Markov Chain replications with ten sets of 500,000 draws[2]

Before estimation, we fixed the quarterly discount rate β for a steady state low interest rate of ***. The steady state gross interest rate, $\overline{R}$, is simply $1/\beta$. The tax parameters for Japan, τ_0^w and τ_0^c are set at 0.35 and 0.07; for the United States, the corresponding values are 0.3 and 0.05. The shares of consumption and government spending to GDP, are 0.6 and 0.4, respectively; for the United States, they are 0.7 and 0.3. The consumption shares match the actual data for the estimation interval. In the absence of investment in this model, the government spending ratio is the remaining share for each country. The steady-state labor income ratio is 1 since labour is the only factor of production.

For estimating the model, we make use of three observables from Japan and the United States: the quarterly rate of growth of GDP, inflation, and the nominal interest rate. All variables were de-meaned.

Table 1: Bayesian Estimation of Parameters and Volatilities

	Distribution			Japan				USA			
Parameters		Priors		Posteriors				Posteriors			
		Mean	Std	Mode	Mean	5%	95%	Mode	Mean	5%	95
ρ_z	Beta	0.5	0.2	0.899	0.593	0.265	0.946	0.773	0.746	0.665	0.831
ρ_g	Beta	0.5	0.2	0.231	0.226	0.120	0.324	0.914	0.906	0.869	0.945
ρ_r	Beta	0.5	0.2	0.929	0.922	0.898	0.948	0.940	0.933	0.918	0.949
ϕ_π	Normal	1.5	0.2	1.456	1.466	1.149	1.776	1.471	1.467	1.142	1.781
ξ_p	Beta	0.5	0.2	0.750	0.777	0.706	0.838	0.793	0.805	0.754	0.856
ξ_w	Beta	0.5	0.2	0.369	0.299	0.113	0.465	0.858	0.752	0.455	0.900
ζ	Normal	6	1	5.989	5.922	4.283	7.573	6.084	6.121	4.482	7.768
η	Nomal	2.5	0.2	2.373	2.638	2.127	3.071	2.725	2.745	2.440	3.056
ϖ	Beta	0.5	0.2	0.738	0.424	0.105	0.770	0.717	0.640	0.396	0.892
Shocks		Priors		Posteriors				Posteriors			
σ_z	Inv.Gamma	0.02	2	0.014	0.024	0.012	0.036	0.025	0.031	0.017	0.044
σ_g	Inv.Gamma	0.02	2	0.028	0.028	0.023	0.032	0.011	0.012	0.009	0.014
σ_r	Inv.Gamma	0.02	2	0.008	0.008	0.007	0.009	0.008	0.008	0.007	0.009

Some comments on the estimates are in order. Starting with the stickiness parameters, the estimated Calvo price coefficient, ξ_p, is similar for both Japan and the United States, while the wage coefficient, ξ_w, is much lower for Japan, with a mean of 0.299, relative to the mean for the United States of 0.752.

[2] All estimation was carried out using the Dynare package developed by M. Julliard available at http://www.dynare.org.

For government spending the persistence coefficient ρ_g is also much lower for Japan than for the United States, with a mean of 0.226 relative to a U.S. value of 0.906, while the volatility for spending shocks is higher in Japan, with a mean of 0.028, relative to a U.S. mean estimate of 0.012. Both countries have a realtively low volatility for the interest rates, while the volatility estimate for the productivity process is relatively higher for the United States, with a mean value of 0.031, relative to a value of 0.024 for Japan.

The other estimated deep coefficients are not markedly different for both countries. The estimates suggest that interest rate shocks are not very important for the period 1990-2008, while government spending shocks are relatively more important for Japan, during the sample period.

3.1 Simulations

To obtain some idea about the likely effects of a tax change, we generated impulse responses for the base case (equations above) and the case allowing for tax-contingent rules. In this latter case, we replace equations (22), (26) and (27) with the following:

$$-\eta c_t + \eta c_{t+1} = r_t - (\pi_{t+1}) - \frac{\overline{\tau^c}}{(1+\overline{\tau^c})}\left[\tau^c_{t+1} - \tau^c_t\right]$$

$$(1+\zeta\varpi)\xi\pi^w_t - (1+\zeta\varpi)\beta\xi\pi^w_{t+1} = (1-\xi)(1-\beta\xi)[\begin{bmatrix} \varpi l_t + \eta c_t - (w_t - p_t) \\ +\frac{\overline{\tau^c}}{(1+\overline{\tau^c})}(\tau^c_t) + \frac{\overline{\tau^w}}{(1-\overline{\tau^w})}(\tau^w_t) \end{bmatrix}$$

$$\begin{bmatrix} \overline{\tau^w}\frac{\overline{WL}}{\overline{PY}}((w_t - p_t) + l_t + \tau^w_t) + \overline{\tau^c}\frac{\overline{C}}{\overline{Y}}(c_t + \tau^c_t) \\ +\frac{\overline{B}}{\overline{PY}}(b_t - p_t) \end{bmatrix} = \begin{bmatrix} \frac{\overline{G}}{\overline{Y}}(g_t) \\ +\frac{\overline{B}}{\overline{PY}}((b_{t-1} - p_{t-1}) - \pi_t + r_{t-1}) \end{bmatrix}$$

Figures 1 and 2 picture the impulse response paths for the model with the modal values of the parameters estimated for Japan and the USA. We also simulate the model with counterfactual debt-contingent rules for tax rates on income. Since consumption tax rates are very low for these countries, we assume that the most likely tax rate to adjust would be the tax rate on income.

We see that the Japanese parameter configuration shows that the state-contingent tax cuts are very effective in reducing the debt/GDP ratio. One reason, of course, is that the government spending process has a much lower autoregressive coefficient, and wages are more flexible, than the US government spending and wage-setting process. Thus, the tax cuts, induce more labor supply and more revenue, and that together with the fall in government spending, induces a more rapid fall in the Debt/GDP ratio for Japan comapred to the United States.

Both parameter configurations show that the consumption response to the spending is very flat and that real interest rise whether or not there are state-contingent tax rates, and that the initial change may be greater or less, under the state-contingent tax rates, relative to the base case.

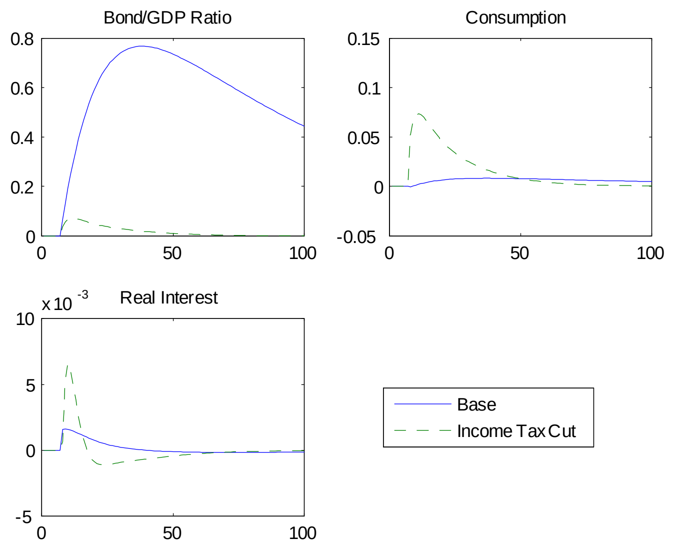

Figure 1: Base Simulation and Debt-Contingent Income Taxes: Case for Japan

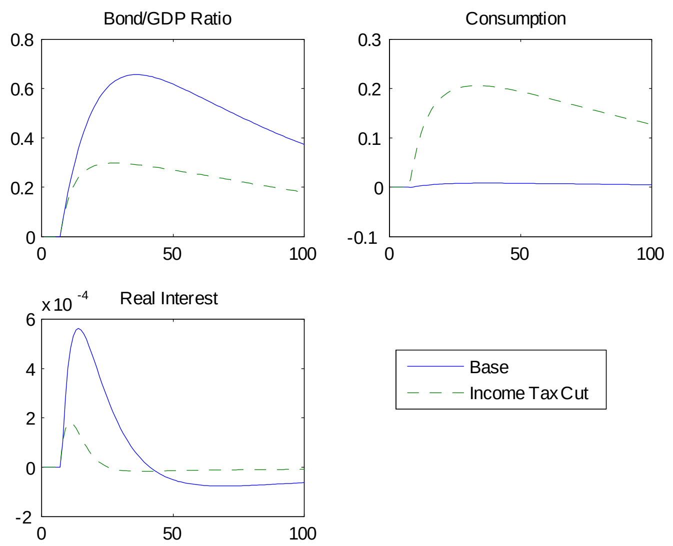

Figure 2: Base Simulation and Debt-Contingent Income Taxes: Case fot the USA

4 Asymmetric Rules

In the previous section, we worked with small changes and symmetric rules. In reality, fiscal rules are likely to be asymmetric - they are more likely to be reduced than raised. In this section, we simulations allow for asymmetric rules. To do so we employ the PEA algorithm and work with a representative country.

4.1 Calibration

The model is calibrated and rely on typical specifications used in the literature with due reference to Japan and the USA. Table 1 gives the values of the parameters which determine the steady state values of the model, as well as those determining the dynamics of the model including the stochastic process. Note we specify two values of the discount parameter β. The first is for a high interest rate econommy, with an annualized risk free rate of return equal to 5%, while the high β corresponds to a low interest rate economy, with an annualzied risk free rate of .4%.

Table 2: Calibrated Values

Symbol	Definition	Value
	Parameters Governing the Steady State	
α	Prof Coeff-L	1
β	Discount Rate	0.9879, 0.999
ϖ	Elasticity of Labor Supply	1
η	Relative Risk Aversion	3
τ_0^w	Labor Income Tax	0.2
τ_0^c	Consumption Tax	0.1
	Parameters for Dynamics and Stochastic Process	
ξ_w, ξ_p	Calvo Wage and Price Coefficient	0.85
ζ	Demand Elasticity for Goods & Labour	6
ρ_G	Spending Smoothing Coefficient	0.9
ρ_R	Taylor Smoothing Coefficient	0.75
ϕ_π	Taylor Inflation Coefficient	1.5
ϕ_B	Debt-Contingent Interest Coefficient	-.1
τ_w, τ_c	Debt-Contingent Tax Coeff	-.1
ρ_w, ρ_c	Tax Rate Smoothing Coefficient	.9
σ_G	Std. Deviation	.01

4.2 Steady-State Solution

$$1 = \beta(1+R) \tag{28}$$
$$PG = \tau_0^w WL + \tau_0^c PC \tag{29}$$
$$Y = \alpha L \tag{30}$$
$$Y = C + G \tag{31}$$
$$L^{-\varpi}(1+\tau_0^c)P = C^{-\eta}(1-\tau_0^w)W \tag{32}$$
$$W = \alpha P \tag{33}$$

Given the parameter configuration, the steady state of the endogenous variables of the model comes from solving the following system of equations, predicated on the assumption that there is no outstanding public debt in the steady-state, $B = 0$ and where the steady state price level, P, is normalised at unity The first steady state condition tells us that the steady state, post-tax gross risk free return, should be equal to the inverse of the social discount rate β. For the government sector a balanced budget means that the tax revenue just covers the government expenditure. In the steady state, goods market equilibrium for the closed economy requires that production of goods be equal to the demand for consumption and government goods. For the labor market, the marginal disutility of labor should be equal to the productivity of labor, net of taxes, times the marginal utility of wealth. The following steady state values come from the system of nonlinear equations.

Table 3: Steady-State values

Symbol	Definition	Value
B	Bonds	0
C	Consumption	0.8528
G	Government Spending	0.3198
L	Labor	1.1726
P	Prive Level	1
R	Annualized Interest Rate	0.05, 0.004
W	Wage Rate	1
Y	Output	1.1726

4.3 Solution and Accuracy Assessment

For solving the model, we make use of the Parameterized Expectations Algorithm (PEA).. The model has five forward-looking variables, for consumption C, for the numerator and denominator of the Calvo pricing equation, N_t and D_t, as well as for the numerator and denominator of the wage setting equation given by A_t^{w1}, A_t^{w2}.Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to parameterize decision rules for C_t, N_t, D_t, A_t^{w1}, A_t^{w2},with nonlinear approximations or functional forms ψ^C, ψ^N, ψ^D , ψ^{A1}, ψ^{A2} which minimize the Euler equation errors for consumption represented by equations (6) , as well as for the Calvo pricing and wage dynamics, given by equations (**??**) and (**??**

$$C_t = \psi^C(\mathbf{x}_t; \Omega_C) \tag{34}$$
$$N_t = \psi^N(\mathbf{x}_t; \Omega_N) \tag{35}$$
$$D_t = \psi^D(\mathbf{x}_{t-1}; \Omega_D) \tag{36}$$
$$A_t^{w1} = \psi^{A1}(\mathbf{x}_t; \Omega_{A1}) \tag{37}$$
$$A_t^{w2} = \psi^{A2}(\mathbf{x}_t; \Omega_{A2}) \tag{38}$$

The symbols Ω_C, $\Omega_N, \Omega_D, \Omega_{A1}$, and Ω_{A2} represent the parameters for the expectation function, while ψ^C, ψ^N, ψ^D, $\psi^{A1}$and ψ^{A2} are the expectation approximation functions. The symbol $\mathbf{x}_t$ represents a vector of observable state variables known at time t: namely the three state variables: government spending, G_t,government bonds, B_{t-1}, and the interest rate R_{t-1}, as well as the two state-contingent tax rates, τ_t^w, τ_t^c

$$\mathbf{x}_t = [\ln(G_t), B_{t-1}, R_{t-1}, \tau_t^w, \tau_t^c] \tag{39}$$

Judd (1996) classifies this approach as a "projection" or a "weighted residual" method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). The functional forms are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994) or neural networks (Duffy and McNelis (2001)). We use a neural network specification with one neuron for each of the decision variables. The neurons take on values between [0, 1] for a logsigmoid function and between [-1, 1] for a tansigmoid function The functions were then weighted by coefficients, and an exponent or anti-log function applied to the final value. The functions were multiplied by the steady state values to ensure steady state convergence.

In the algorithm, non-negativity constraints were imposed for all of the variables. We also applied the usual no-Ponzi game to the evolution of real government debt namely:

$$\lim_{t \to \infty} B_t \exp^{-it} = 0 \tag{40}$$

The accuracy of the simulations may be checked by the Judd-Gaspar statistic which is the maximum value of the absolute value of the Euler equation error for consumption C, Calvo price P, and the Calvo wage W. That is, for realization j, with size T, the accuracy measure is:

$$\begin{aligned}
JG_{\max}^{(j)} &= \max \left[\frac{|\nu_{Ct}|}{C_t}, \frac{|\nu_{Pt}|}{P_t} \frac{|\nu_{Wt}|}{W_t} \right] \\
\nu_{Ct} &= \left(\frac{C_t^{-\eta}}{P_t(1+\tau_t^c)} - \beta \frac{C_{t+1}^{-\eta}}{P_{t+1}(1+\tau_{t+1}^c)}(1+R_t) \right) \\
\nu_{Pt} &= \left(P_t^o - \frac{Y_t \left(P_t\right)^{\zeta} A_t + \beta \xi N_{t+1}}{Y_t \left(P_t\right)^{\zeta} + \beta \xi D_{t+1}} \right) \\
\nu_{Wt} &= \left((W_t^o)^{1+\zeta\varpi} - \frac{(W_t)^{\zeta+\zeta\varpi} \left(L_t^{1+\varpi}\right) + \xi\beta.A_{t+1}^{w1}}{\Lambda_t(1-\tau_t^w)\left(W_t\right)^{\zeta} L_t + \xi\beta.A_{t+1}^{w2}} \right)
\end{aligned}$$

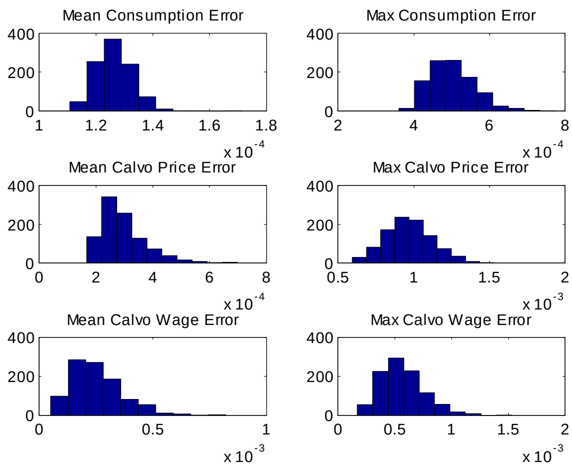

Figure 3: Distribution of Mean and Maximum Judd-Gaspar Errors for Base Simulation

We solved and simulated the model for $T = 500$, for 1000 realizations of the stochastic process governing G_t.Figure gives the mean and maximum values for each maximum error over the 1000 experiments. We see that the decision rules give a high degree of accuracy. Both the mean and the maxima of the maxima absolute Euler equation errors is quite small, well less than one perent.

4.4 Simulation Results

4.4.1 Base Simulation

We first analyze the base simulation for the model, with no state contingent taxes, for a large (25%) government spending shock. Then we take up the alternatives of state-contingent taxes.

Figure 4 pictures government spending and consumption (as percentages of their steady-state values) , the real government bond/real GDP ratio for a 25% shock to government spending, and the real interest rate. We see that the rise in spending dies out after 20 quarters but the real debt overhang continues (although it does converge to a zero steady state). What is revealing is that there is there is practically no change in consumption. For a 25% increase in government spending, the maximum increase in consumption is less than .1%. If the tax rates do not change and the monetary authority maintains its inflation targeting stance, the consumption effect of a spending increase is very non-existent. We also see that the debt expansion brings with it an increase in real interest rates. This is a recurring result, which does not change when alternative tax rules are used [see Ganelli (2007) : p. 1024].

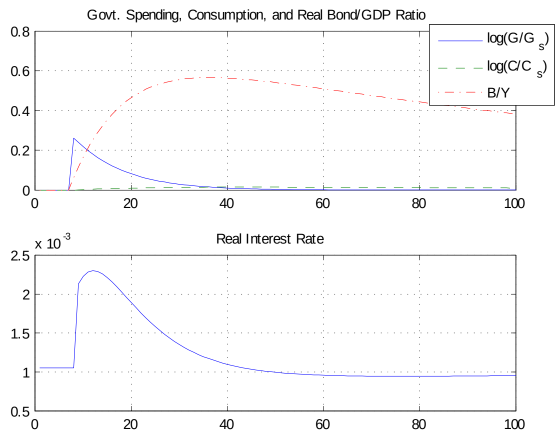

Figure 4: Base Simulation: Government Spending, Consumption, Real Bond/GDP Ratio, and the Real Interest Rate

4.4.2 Debt-Contingent Interest Rate Rule

Can the debt overhang be reduced more quicly , with a debt-contingent interest-rate rule? Figure 5 pictures the response of bonds, consumption, and the real interest rate under the base simulation and under state-contingent interest-rate rule, with the parameter $\phi_B = -.1$. Figure 5 shows that the bond overhang falls after about 15 quarters. The consumption stimulus of the 25% public spending increase is higher than in the base case, due to the fall in the real interest rates. The real interest rate initially rises as much as the base case, but then falls more rapidly, below its steady state. This fall in the real interest rate leads to the rise in consumption and tax revenue which in turn leads to a fall in the debt/gdp ratio.

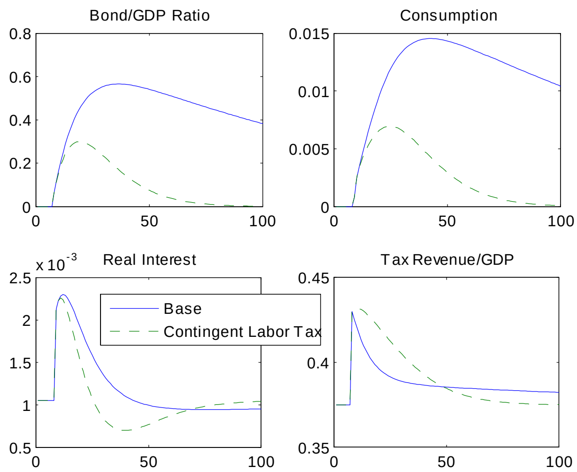

Figure 5: Bonds, Consumption, and Real Interest under Base Simulation and Debt-Contingent Interest Rates

4.4.3 Debt-Contingent Income Taxes

Figure 6 pictures the response of the bonds, consumption, the real interest rate when the state contingent taxs cuts were imposed labor income. A similar pattern emerges, but with stronger effects the debt/gdp ratio is greatly reduced, while consumption rises, this time reaching a maximum of almost 20% relative to its steady state.

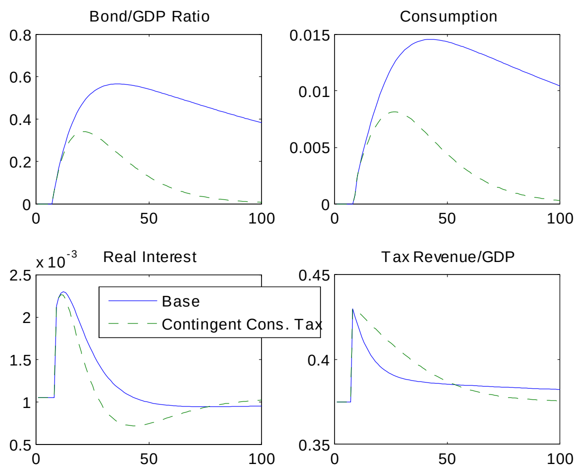

Figure 6: Bonds, Consumption, and Real Interest under Base Simulation and Debt-Contingent Income Taxes

4.4.4 Debt-Contingent Consumption Taxes

Figure 7 shows the adjustment of the same variables with a state-contingent consumption tax cut. We see a similar pattern as in the case of the income-tax cut. In both cases there is an increase in the real interest rate, but the negative effect of the rise in the real intersest rate on consumption is more than offset by the fall in the tax rates.

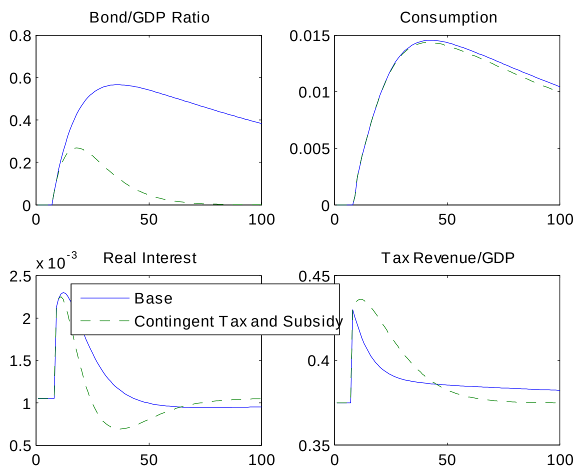

Figure 7: Bonds, Consumption, and Real Interest under Base Simulation and Debt-Contingent Consumption Taxes

4.5 Parameter Robustness: Variations in the Frisch Labor Supply Elasticity

To assess the robustness of our results we evalute the effects of debt-contingent income taxes uder two alternative values fo the Frisch labor supply elasticity, ϖ, first at realtively high value of 2 and a realtively low value of .2. Most calibrated models use values within this range, Clinton, Kumhof, Laxton and Mursula (2011) set this parameter at .5 in their multicountry model.

Figures 8 and 9 picture the paths of the Bond/GDP ratio, Consumption and the Real Interest under the high and low labor supply elasticity assumptions.

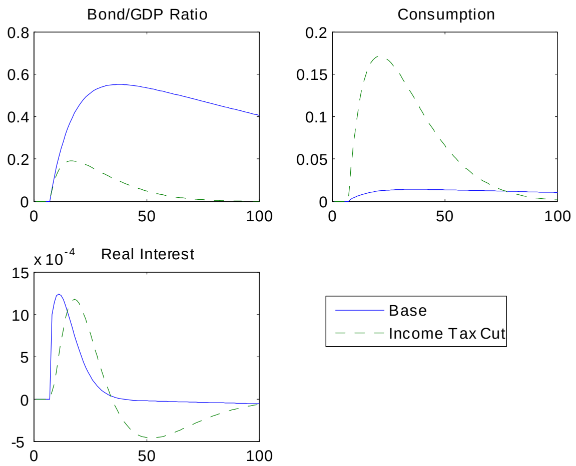

Figure 8: Bonds, Consumption, and Real Interest under Base Simulation and Debt-Contingent Income Taxes with High Labor Supply Elastcity: $\varpi = 2$

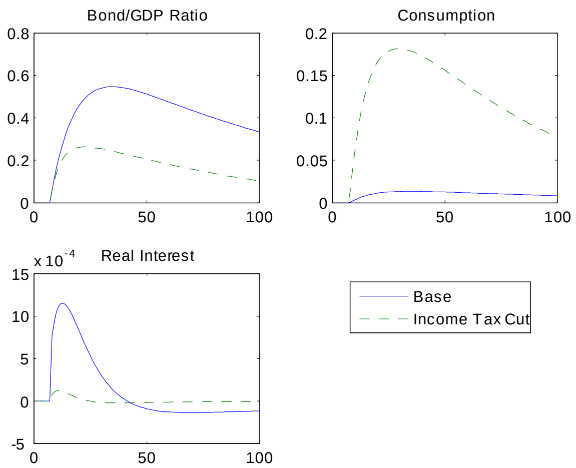

Figure 9: Bonds, Consumption, and Real Interest under Base Simulation and Debt-Contingent Income Taxes under Low Labor Supply Elasticity: $\varpi = .2$

In both cases, the use of the debt-continget tax cuts are effective. In the case of the high labor supply elasticity, the bond reducting is slightly faster, and the real interest rate becomes negative during the adjustment process. But variations in the specification of this parameter make within a range of [.2 2] do not appear to affect the robustness of the results.

5 Conclusion

This paper has shown that the use of debt-contingent tax cuts on labor income, or consumption are effective ways to reduce debt, by stimulating labor income, consumption demand and tax revenue. These instruments are powerful and of course particularly useful when the interest rate is near or at the lower bound. This result is important because Piergallini and Rodano (2009), have warned that households' participation constraints and Laffer-type effects may render fiscal policy rules unfeasible.. For any given target inflation rate, they find that there exists a threshold level of public debt beyond which monetary policy independence is no longer possible. In this case, the dynamics of public debt must be controlled by lower interest rates. But what if interest rates are very near the lower bound? We show that in the case of the zero lower bound, credible, temporary state-contingent tax-rate decreases do indeed increase current and future tax revenues, and thus reduce debt by stimulating consumption and labor income. To be sure, the autoregressive government spending, after the initial shock, is on a downward trajectory, so that the deficit is being reduced, slowly, when the endogeneous tax rates are implemented.

We emphasize that we are not advocating the use of tax cuts when the public debt is out of control. Each of these tax cuts took place after the expenditure surge was reversed, and government spending was falling back towards its steady state. So in our case the budgetary process was on a path of consolidation and stabilization.

Our result contrasts with arguments for a wealth tax, in which taxation becomes an increasing function of public debt, rather than the usual tax on labor or consumption.. Ganelli (2007) has shown that such a tax, in an overlapping generaitons model might not be enough to ensure a well-defined steady-state equilibrium.

As Sims (2009) has noted, low interest rates can be used to reduce the interest expense item in the federal budget and thus reduce debt expansion. If the fiscal authority has control over all aspects of its balance sheet, including interest rates, then we have a regime of fiscal dominance. Aizenman and Marion (2009) have documented several cases in U.S. experience over the past 50 years.. A key implication of our model simulations is that there is no need for the fiscal authority to try to dominate monetary policy when interest rates are above the zero bound. The fiscal authority can implement rules for temporary tax-rate changes which can be effective both at the zero lower bound, and as an alternative to fiscal dominance.

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