Stock Options and Credit Default Swaps:

A Joint Framework for Valuation and Estimation

presented by

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based on joint work with

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Hedge CDS with Stock Options

- The classic Merton (1974) model predicts a positive link between
  - (1) corporate bond credit spreads and (2) stock return volatility
- Many studies have empirically identified this positive link at both the aggregate and firm level (a very long list...)
- When a company defaults, the stock price drops by a sizable amount.
  - The possibility of default generates negative skewness in the stock risk-return return distribution.
- Cremers, Driessen, Maenhout, and Weinbaum (2004): CDS spreads are positively correlated with both stock option implied volatility and the implied volatility skewness across moneyness.
- The positive link has long been recognized in the industry and has been used to justify a cross-market hedge: Hedge credit insurance with stock options.

A Joint Valuation Framework

- We propose a joint framework for valuing stock options and credit default swap spreads written on the same company:
  - Default is controlled by a Poisson process, with stochastic arrival rate.
  - When default occurs, the stock price drops to zero.
  - Prior to default, the stock price follows a continuous process with stochastic volatility.
  - Prior to default, the default arrival rate and the stock variance rate follow a bivariate continuous Markov process.
  - Their joint dynamics are specified to capture the empirical evidence on stock option prices and credit default swap spreads.
- Analytical solutions for stock options and CDS spreads are obtained.

Joint Estimation

- We estimate the joint dynamics of default arrival and stock variance rates
- Using both stock option prices and CDS spreads
- Via maximum likelihood (ML) with unscented Kalman filter (UKF):
  - Given the model parameters, the UKF extracts the conditional joint density of the default arrival and stock return variance rates.
  - Given the conditional density, ML estimates the model parameters.
- Bottom line:
  Parameters are fixed over time. Only the state variables are allowed to vary.

Literature "R" Us

- Merton (1974): Asset value follows GBM with constant volatility.
  $\Rightarrow$ *Important in providing the link, but the link is too strong.*

- Modifications: Hull, Nelken, White (04), Cremers, Driessem, Maenhout (06), ...
  $\Rightarrow$ *Static calibration with no dynamic consistency.*

- "Jump-to-default" equity models: Davis&Lischka(99), Carayannopoulos&Kalimipalli (03), Carr&Linetsky (06), Das&Sundaram(06), ...

- The advantages of our modeling and estimation approach:
  (1) Correlated but separate dynamics for stock variance and default arrival generate imperfect correlation between CDS and option implied volatilities.

  (2) Dynamic consistency: Dynamics are specified and risks are priced.

  - *Static consistency*: Prices are consistent cross-sectionally at a fixed point in time – sufficient for market makers not holding overnight positions.
  - *Dynamic consistency*: Prices are also consistent over time — important for hedge funds holding long-term positions.

Joint Dynamics

- Company default follows a Poisson process, with stochastic arrival rate λ_t.
- Prior to default, the risk-neutral $\mathbb{Q}$-dynamics of the company's stock price:

$$dP_t/P_t = (r_t - q_t + \lambda_t)\,dt + \sqrt{v_t}dW_t^P, \leftarrow \textit{stochastic mean and variance}$$

- Joint (*but separate*) dynamics of stock variance and default arrival rates:

$$\begin{aligned} dv_t &= (\theta_v - \kappa_v v_t)\,dt + \sigma_v\sqrt{v_t}dW_t^v, \leftarrow \textit{stochastic stock return variance } (v_t) \\ \lambda_t &= \beta v_t + z_t, \leftarrow \textit{default arrival comoves } (\beta) \textit{ with stock variance} \\ dz_t &= (\theta_z - \kappa_z z_t)\,dt + \sigma_z\sqrt{z_t}dW_t^z, \leftarrow \textit{independent credit risk movements } (z_t) \\ \rho &= \mathbb{E}\left[dW^P dW^v\right]/dt, \leftarrow \rho < 0 \Leftrightarrow \textit{“leverage effect.”} \end{aligned}$$

- Extensions on default dynamics:
$\lambda_t = \beta^\top F_t + z_t$, with $F_t = [v_t, \ln P_t, r_t]^\top$, and z_t can be multi-dimensional.

Joint but Separate Dynamics

- To summarize, we allow separate (albeit correlated) dynamics between stock returns, stock variance, and default arrival rates.

- Our specification can capture the following empirical correlations:
  - Stock returns and increments in return variances are negatively correlated.
  - Stock returns and increments in default arrival rates are negatively correlated.
  - Increments in stock variance rate and in default arrival rates are positively correlated.

- Our specification also recognizes the imperfections of their co-movements:
  - *Stock price, stock variance, and CDS spreads all have their own independent movements.*
  - *CDS hedging requires multiple instruments.*

Valuing Stock Options

- The value of a European call option

$$
\begin{aligned}
c(P_t, K, T) &= \mathbb{E}_t\left[\exp\left(-\int_t^T (r(s)+\lambda(s))\,ds\right)(P_T-K)^+\right] \\
&= B(t,T)\,\mathbb{E}_t\left[\exp\left(-\int_t^T \lambda(s)\,ds\right)(P_T-K)^+\right],
\end{aligned}
$$

where $B(t,T)$ is the time-t discount factor with maturity date T.

- The expectation can be solved by inverting the discounted generalized Fourier transform,

$$
\begin{aligned}
\phi(u) &\equiv \mathbb{E}_t\left[\exp\left(-\int_t^T \lambda(s)\,ds\right)e^{iu\ln(P_T/P_t)}\right], \quad u \in \mathcal{D} \subset \mathbb{C}, \\
&= \exp\left(iu(r(t,T)-q(t,T))\tau - a(\tau) - b(\tau)^\top x_t\right), \quad \tau = T-t,
\end{aligned}
$$

where the affine coefficients $(a(\tau), b(\tau))$ can be solved **analytically** as a function of the risk-return factor dynamics.

Valuing Credit Default Swap Spreads

- The present values of the premium and protection legs of a CDS contract (assuming continuous payoff) are:

$$\begin{aligned}\text{Premium}(t,T) &= \mathbb{E}_t\left[S(t,T)\int_t^T \exp\left(-\int_t^s (r(u)+\lambda(u))du\right)ds\right],\\ \text{Protection}(t,T) &= \mathbb{E}_t\left[w\int_t^T \lambda(s)\exp\left(-\int_t^s (r(u)+\lambda(u))du\right)ds\right],\end{aligned}$$

  with $(1-w)$ denoting the recovery rate, and $S(t,T)$ denoting the "CDS spread" for a contract that starts at t and expires at T.

- Under our model, we can solve for the values of both legs **analytically**:

$$\begin{aligned}\text{Premium}(t,T) &= S(t,T)\int_t^T B(t,s)\exp\left(-a_\lambda(s-t)-b_\lambda(s-t)^\top x_t\right)ds,\\ \text{Protection}(t,T) &= w\int_t^T B(t,s)\left(c(s-t)+d(s-t)^\top x_t\right)e^{\left(-a_\lambda(s-t)-b_\lambda(s-t)^\top x_t\right)}ds.\end{aligned}$$

- In estimation, the integrals are discretized to quarterly premium payments.

Data

- CDS and stock options on four companies:

  Ford (F), General Motors (GM), Altria Group (MO), Duke Energy (DUK).

- CDS spreads from Bloomberg:

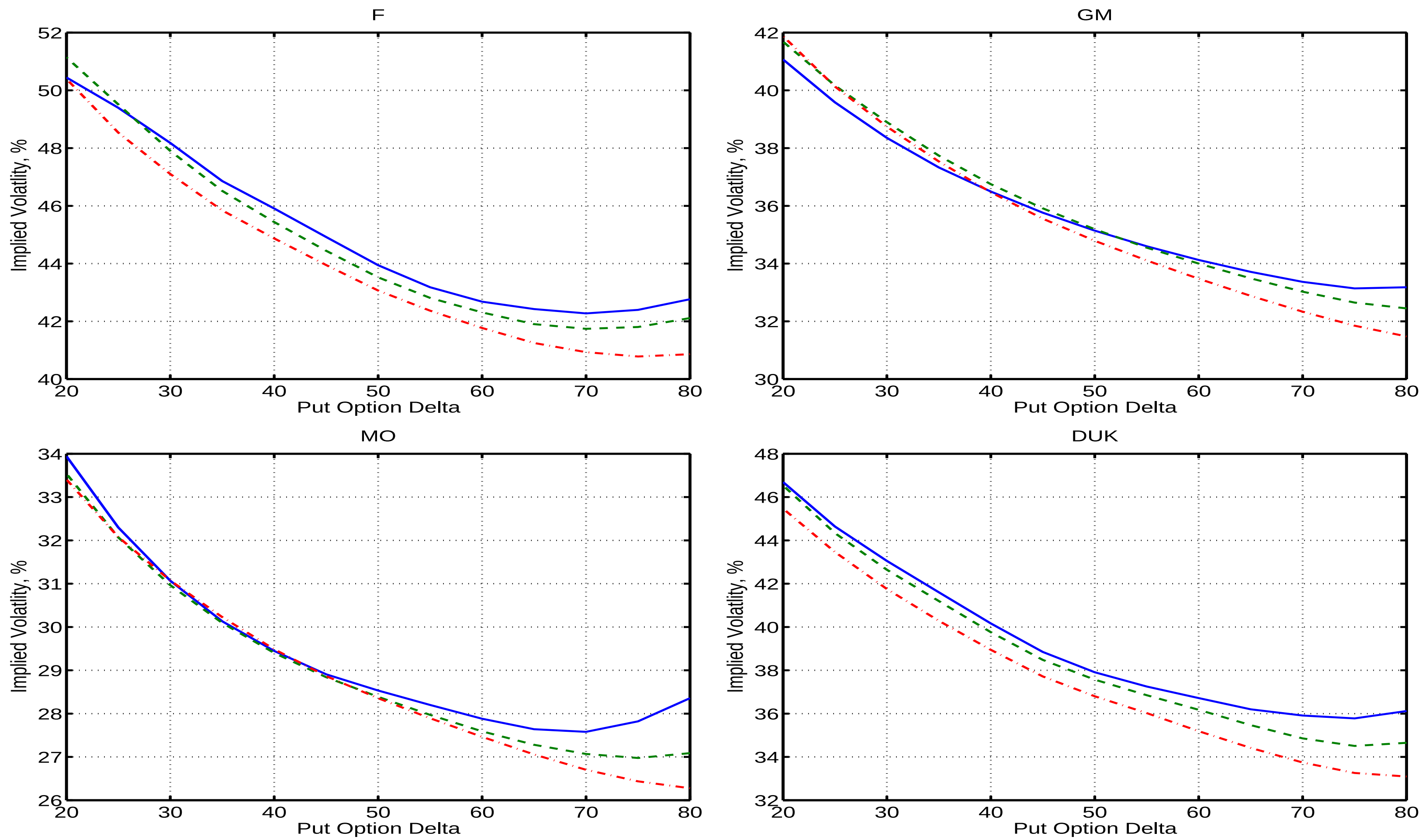
  Five fixed maturities: 1, 3, 5, 7, 10 years.

- Stock options from OptionMetrics:
  Standardized implied volatility surface
  - From 20 to 80 delta, every 5 delta. At each delta, we average the given implied volatilities of calls and puts.
  - Three fixed option terms: 1, 2, and 3 months.

- We convert the implied volatility into out-of-the-money European option prices in percentages of the underlying spot.

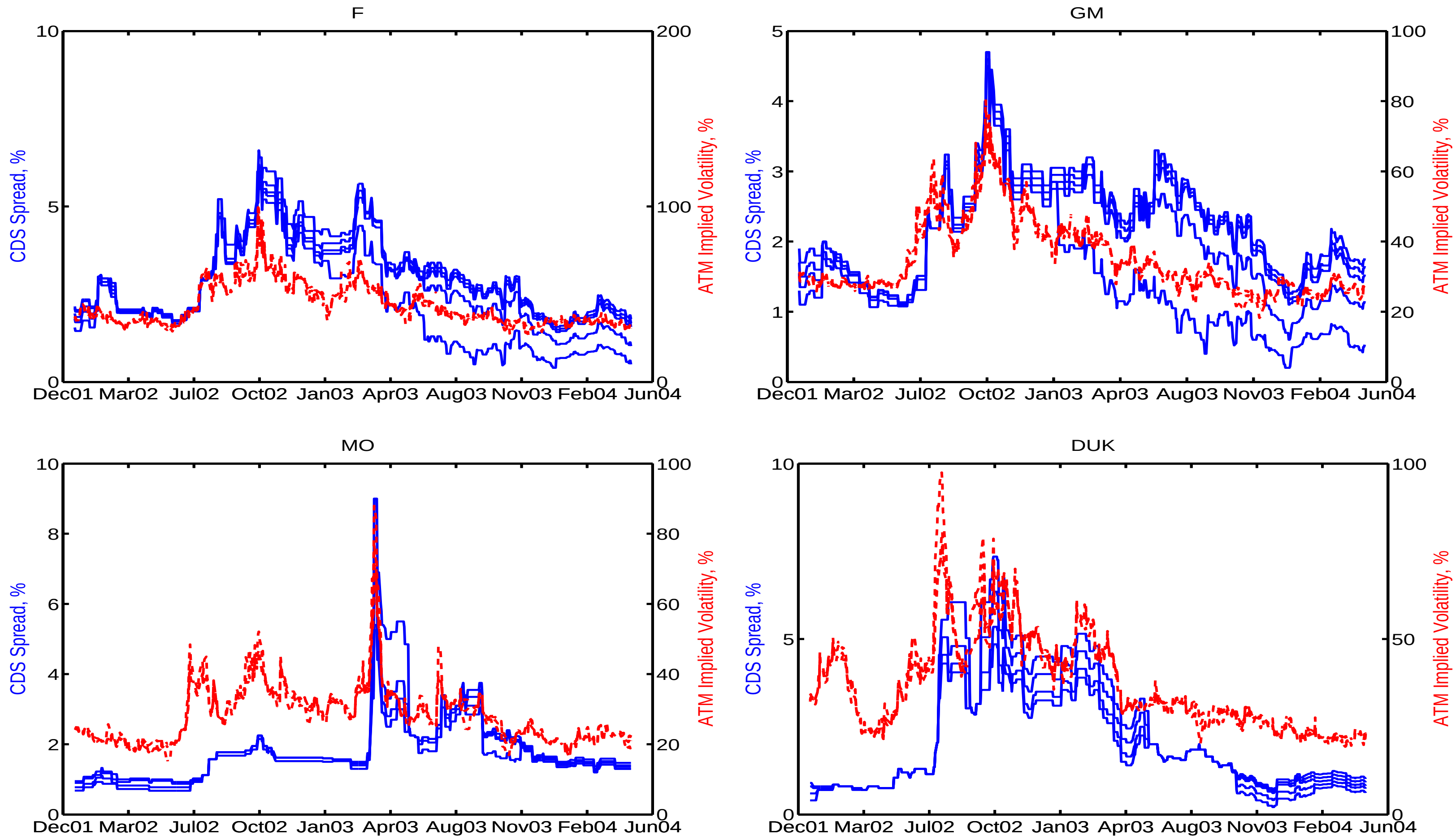
- Sample period: January 2, 2002 to April 2004 (122 weekly observations).

Average Implied Volatility Smirk



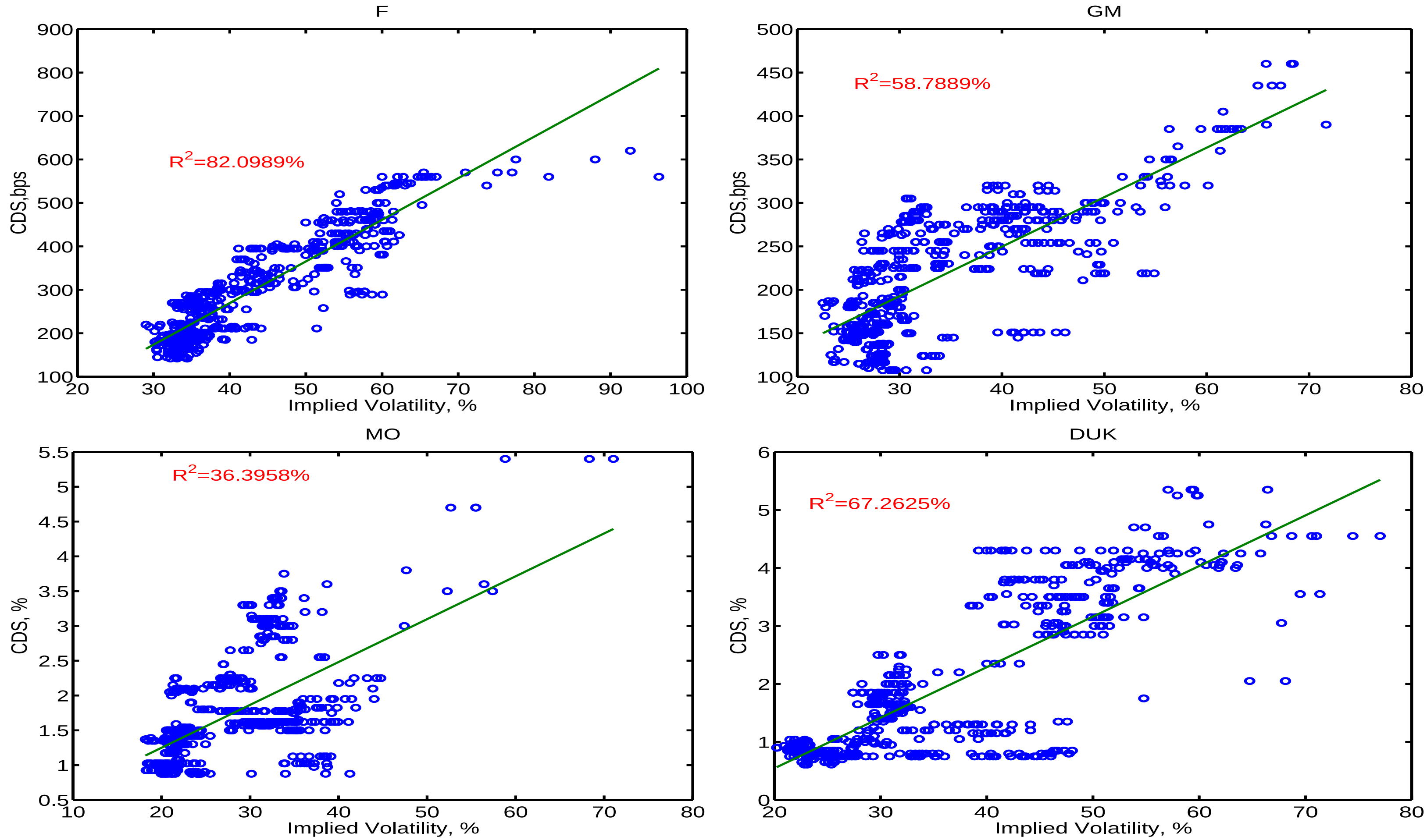
Negative skewness in risk-neutral return distribution $\Leftarrow$ *(1)* $\lambda_t > 0$, *(2)* $\rho < 0$.

Co-Movements of CDS and ATMV



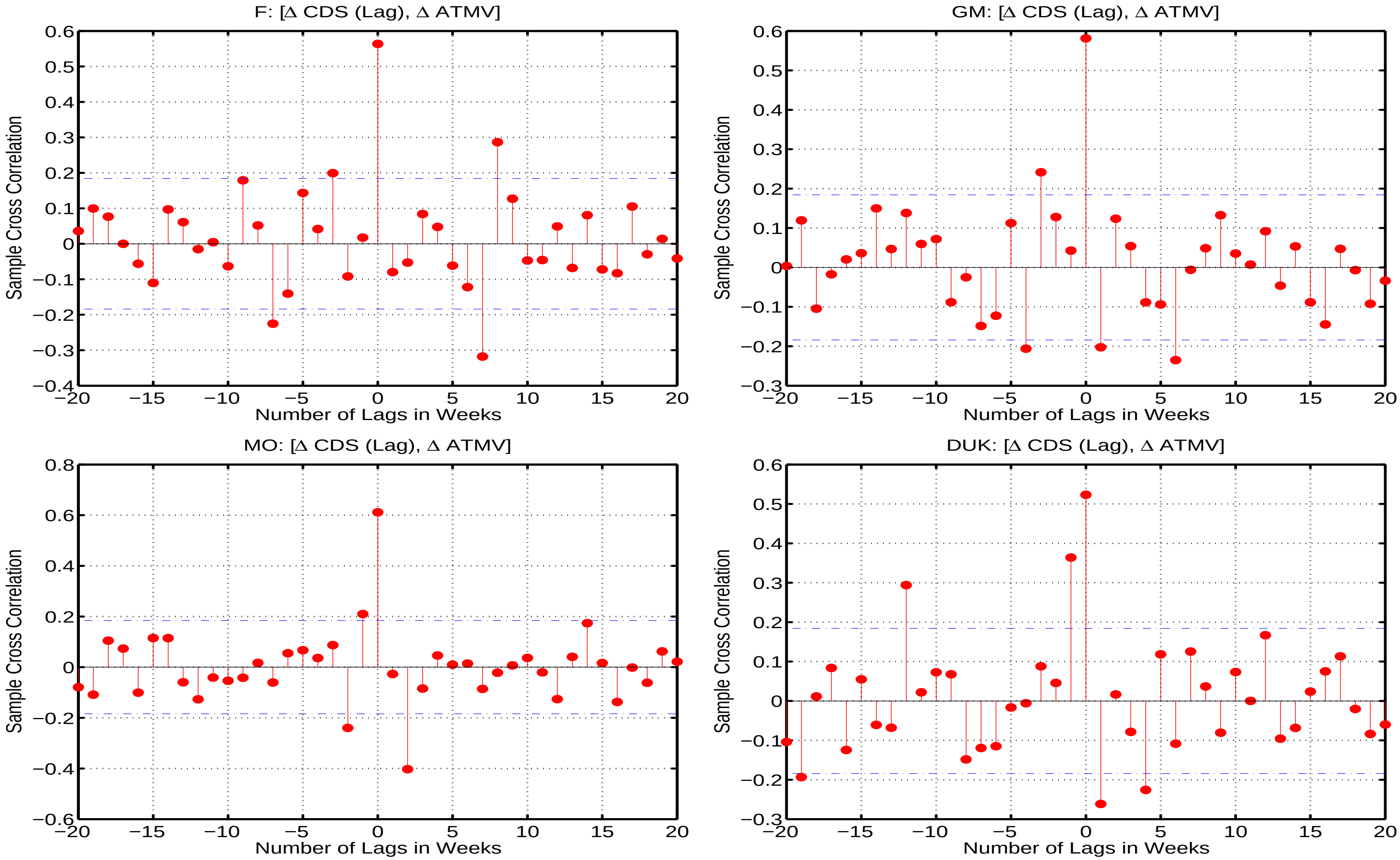
Red — Option implied volatility   Blue — CDS spreads

Regression Evidence



Positive, but imperfect, co-movements $\Leftarrow \beta > 0, z_t$.

Cross-Correlogram of Weekly Changes



Strongly positive contemporaneous correlation $\Leftarrow \beta > 0$.

Estimation Procedure

- We cast the model into a state-space form:
  - State propagation–Euler approximation of the two risk factors $x_t \equiv [v_t, z_t]^\top$:
    $x_t = \theta\Delta t + \varphi x_{t-1} + \sqrt{\beta x_{t-1}\Delta t}\varepsilon_t$.
  - Measurement equations are on CDS spreads S and stock options O:
    $y_t = h(x_t;\Theta) + e_t$, with

$$h(x_t;\Theta) = \begin{bmatrix} S(x_t, t+\tau_S;\Theta) \\ O(x_t, t+\tau_O, \delta;\Theta) \end{bmatrix}, \qquad \begin{array}{l} \tau_S = 1,3,5,7,10 \text{ years} \\ \tau_O = 30,60,91 \text{ days}; \delta = 20,25,\cdots,80. \end{array}$$

- Given model parameters, UKF generates efficient forecasts and updates on conditional mean and covariance of states and measurements sequentially.

- Choose model parameters to maximize the likelihood on forecasting errors.

- The demand for tractability: Thousands of iterations on 122 days, $(5+39)$ prices per day. (In our case, less than 4 seconds per iteration (5,368 prices)).

Pricing Performance on Options

R-squared:

Delta		20	25	30	35	40	45	50	55	60	65	70	75	80
F	1m	0.94	0.94	0.94	0.93	0.93	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.93
F	2m	0.97	0.97	0.97	0.96	0.96	0.96	0.97	0.97	0.98	0.99	0.98	0.98	0.97
F	3m	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97	0.98	0.98	0.97	0.97	0.96
GM	1m	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.99	0.99	0.99	0.97
GM	2m	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.97
GM	3m	0.99	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.96
MO	1m	0.95	0.95	0.95	0.96	0.96	0.97	0.98	0.98	0.98	0.98	0.98	0.95	0.88
MO	2m	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.96	0.92
MO	3m	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.97	0.96	0.96	0.96	0.95
DUK	1m	0.97	0.97	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.96	0.95	0.95
DUK	2m	0.97	0.98	0.98	0.98	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.97	0.96
DUK	3m	0.97	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.97	0.97	0.97	0.96

Pretty good!

Remember: 2 degrees of freedom (v_t, z_t) each day to match all these prices, and more...

Pricing Performance on CDS spreads

R-squared:

CDS	1	3	5	7	10
F	0.85	0.96	0.93	0.91	0.89
GM	0.79	0.76	0.63	0.55	0.50
MO	0.34	0.35	0.30	0.27	0.31
DUK	0.99	1.00	0.99	0.97	0.84

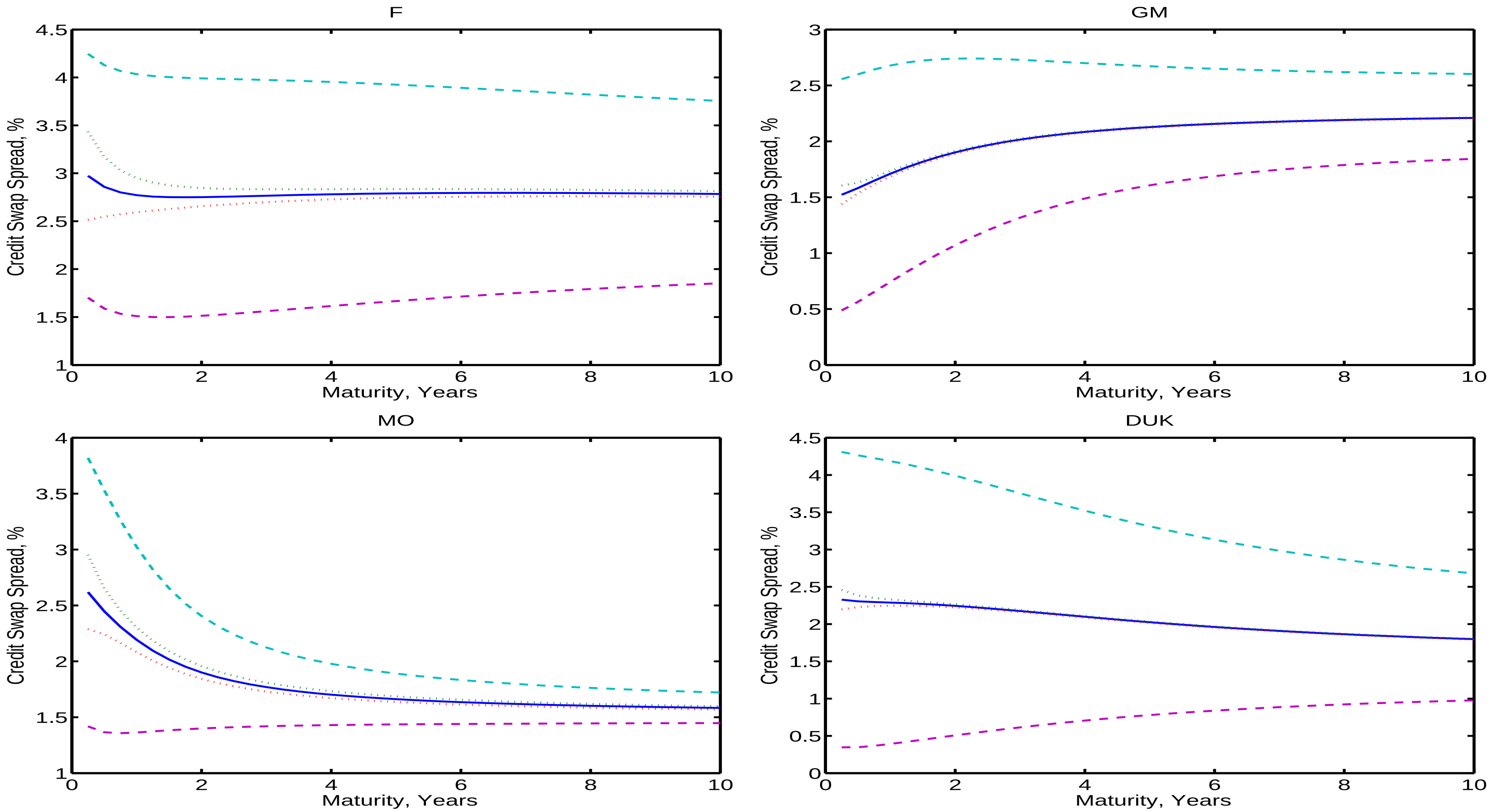
Some remaining tensions on GM and MO.

Factor Dynamics

Companies	F		GM		MO		DUK	
Mean Reversion Speeds under $\mathbb{Q}$ and $\mathbb{P}$:								
κ_v	4.0788	(47.34)	7.8085	(121.24)	5.7515	(163.79)	6.5862	(69.90)
κ_z	0.0067	(0.40)	0.0065	(0.12)	0.0067	(0.08)	0.0485	(2.84)
$\kappa_v^{\mathbb{P}}$	1.1878	(1.52)	1.6451	(25.48)	1.2558	(5.34)	3.4894	(2.46)
$\kappa_z^{\mathbb{P}}$	0.1745	(3.71)	1.8806	(1.81)	0.1811	(0.81)	0.2966	(1.43)

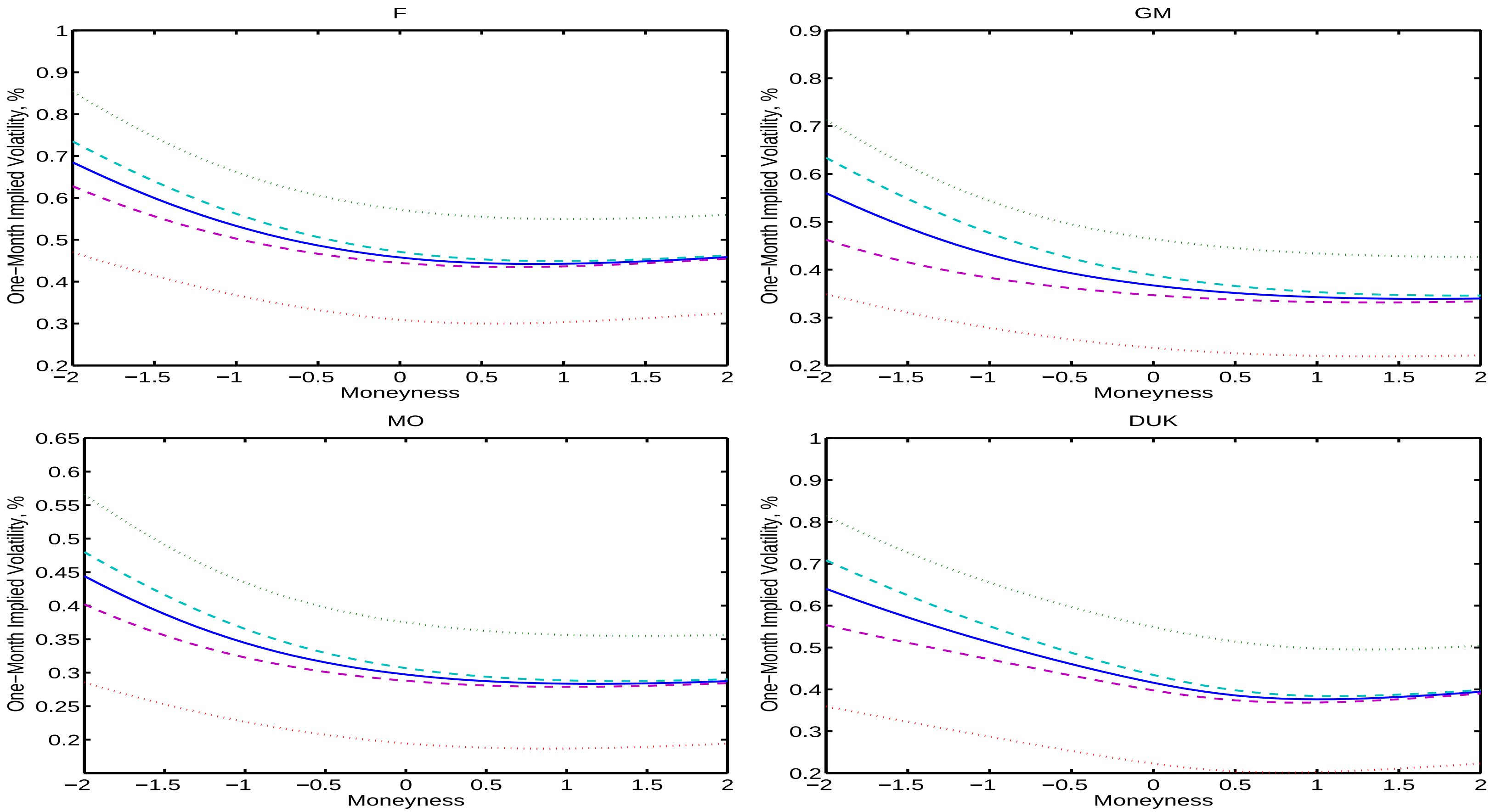
- The credit risk factor z is more persistent than the stock variance rate v, more so under the risk-neutral measure $\mathbb{Q}$ than under the statistical probability measure $\mathbb{P}$.
  - $\mathbb{P}$: *Credit risk variations are more difficult to forecast than stock variance.*
  - $\mathbb{Q}$: *Shocks in stock variance affect the term structure mainly at short maturities. The impacts of the credit risk factor remain at longer maturities.*

Term Structure of CDS Spreads



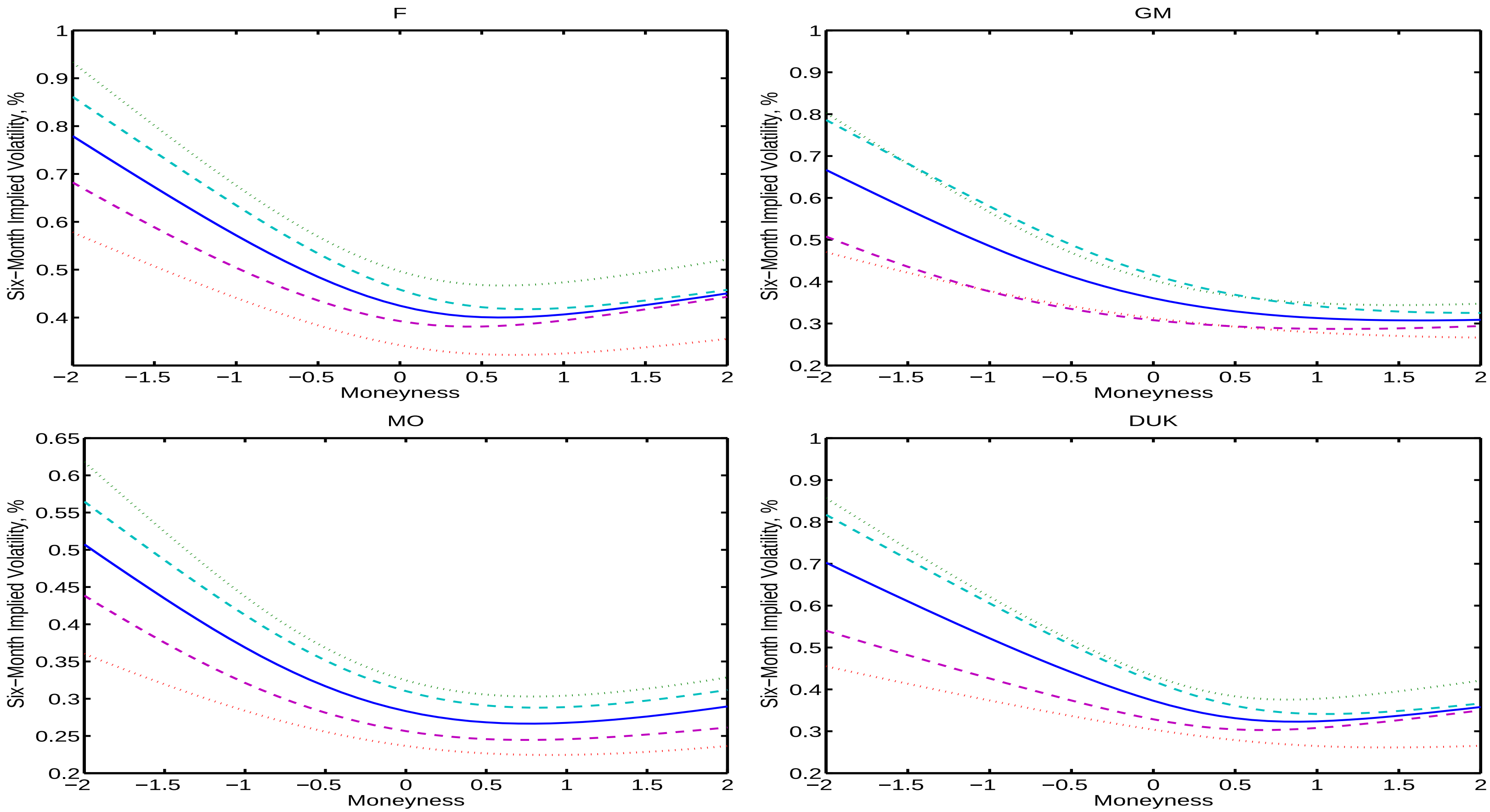
Credit risk (——) dominates, but stock variance (· · ·) also matters ...
mainly at short maturities.

Implied Volatility Smirks: 1 month



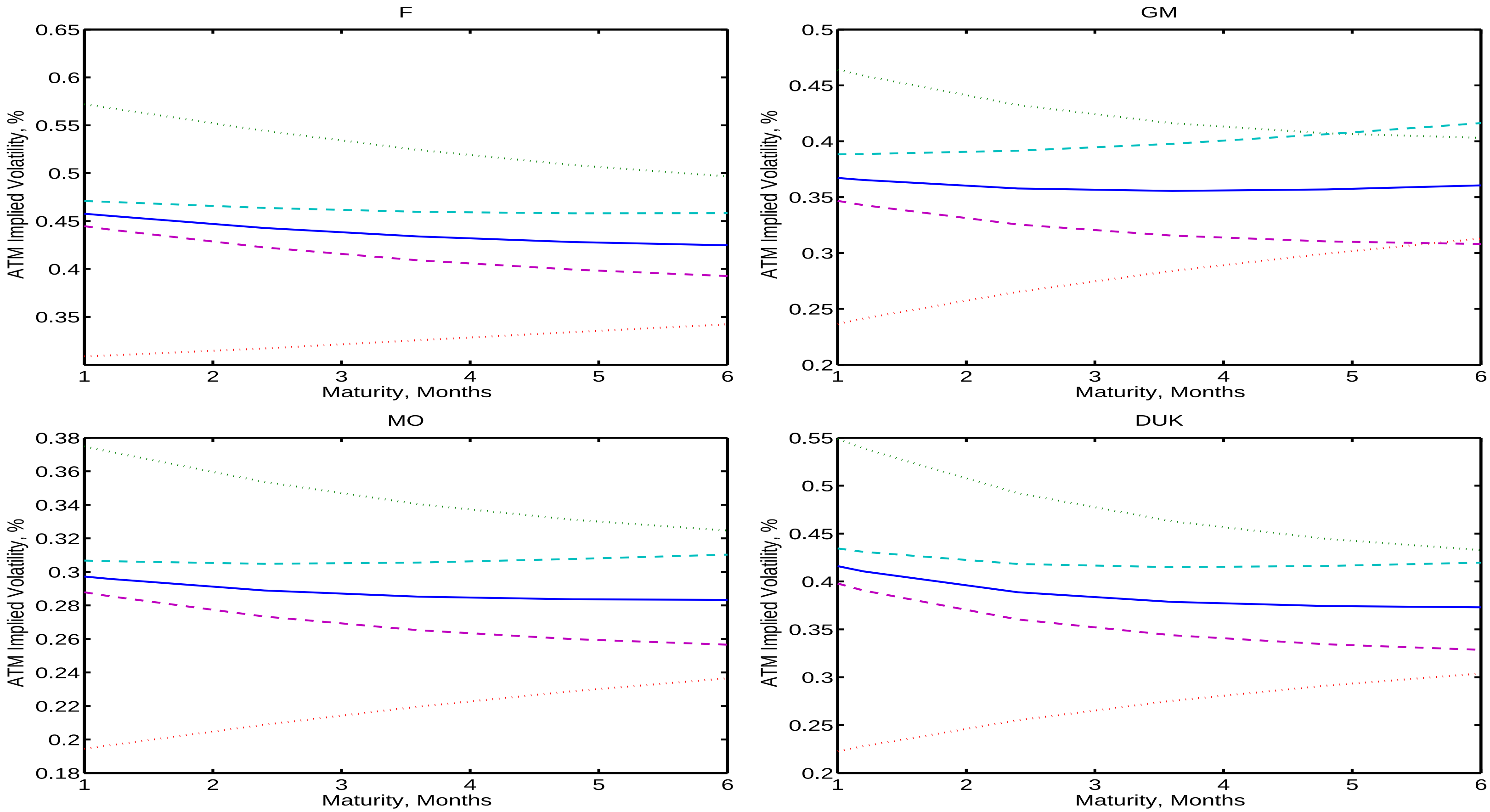
The impact of stock variance is uniform across moneyness, but credit risk matters more at lower strikes.

Implied Volatility Smirks: 6 month



The impact of credit risk on options increases at longer maturities.

Term Structure of ATM Implied Volatilities



At maturity increases, the impact of stock variance declines, but the impact of the credit risk increases.

Market Prices of Factor Risks

$$\gamma_v = (\kappa_v - \kappa_v^{\mathbb{P}})/\sigma_v, \quad \gamma_z = (\kappa_z - \kappa_z^{\mathbb{P}})/\sigma_z.$$

Companies	F		GM		MO		DUK	
γ_v	2.1044	(3.56)	7.1046	(35.35)	5.9851	(17.49)	1.5347	(2.04)
γ_z	-0.9645	(4.20)	-3.2600	(1.80)	-0.1099	(0.71)	-0.6733	(1.20)

- The market price of risk is negative for the credit risk factor z, but positive for the stock variance rate v.

- *Negative variance risk premia observed in other studies are induced by variation in default arrival rates, not by the variation in diffusion induced stock volatility.*

- *In our sample period, selling credit protection generates higher average returns per unit of risk than selling variance through options or through synthetic variance swaps.*

Separately Identifying Default Arrival and Recovery Rate

Companies	F	GM	MO	DUK
$1-w$	0.6417 (85.54)	0.8090 (134.09)	0.4688 (13.89)	0.5657 (126.65)

- The literature has found it difficult to separately identify λ and $(1-w)$ from corporate bond credit spreads or CDS.
- *Using the overlapping information in stock options and CDS spreads is helpful to separate them and identify the recovery with high statistical significance.*
- *The recovery rate estimates during our sample period are high compared to standard assumptions.*

Concluding Remarks

- CDS spreads move together with stock option implied volatilities.
- But for some stocks, their positive correlation is far from perfect.
- We propose a joint valuation and estimation framework that allows **separate** dynamics for the default arrival rate, the stock variance rate, and stock returns, with dynamic interactions consistent with the evidence.
- Compared to daily calibration of models with constant parameters, we explicitly model the factor dynamics that affect option & CDS prices.
- Our maximum likelihood estimation also complies with the dynamic consistency of our theoretical model — State variables are allowed to vary, but parameters are held constant through time.
- Our framework is intended to provide a stable and dynamically consistent foundation for cross-market trading (e.g., capital structure arbitrage)...

Concluding Remarks

- and a consistent framework for pricing equity-credit combo products, e.g., convertibles, equity default swaps (EDS),...