

Real Exchange Rates Over the Past Two Centuries: How Important is the Harrod-Balassa-Samuelson Effect?*

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December 14, 2004

Abstract

Using data for 1820-2001 for the US, the UK and France, we test for the presence of real effects on the equilibrium real exchange rate (the Harrod-Balassa-Samuelson, HBS effect) in an explicitly nonlinear framework and allowing for shifts in real exchange rate volatility. A statistically significant HBS effect for sterling-dollar captures its long-run trend and explains some 40% of its variation. For both real exchange rates there is significant evidence of nonlinear mean reversion towards long-run equilibrium and downwards shifts in volatility corresponding closely to the classical gold standard and Bretton Woods periods.

JEL classification: F31, F41, C1.

Keywords: purchasing power parity; real exchange rate; nonlinear dynamics; Harrod-Balassa-Samuelson effect; productivity differentials.

* *Acknowledgements:* The research reported in this paper was conducted with the aid of research grant from the Leverhulme Trust, which Taylor gratefully acknowledges.

1 Introduction

The purchasing power parity (PPP) level of the exchange rate is the rate of exchange between two national currencies at which one unit of currency will have the same purchasing power in its own country as it will in the country of the foreign currency, once converted into the foreign currency at that exchange rate. Although the actual term “purchasing power parity” was coined less than a hundred years ago (Cassel, 1918), the PPP concept has a very much longer history in economics (see e.g. Grice-Hutchison, 1952; Officer, 1982). It has variously been treated as a theory of short-run or long-run exchange rate determination or, more normatively, as the level at which the authorities should attempt to steer or set the nominal exchange rate in order to maintain international competitiveness. With the experience of two decades of floating exchange rates among the major industrialised countries after the breakdown of the Bretton Woods system in the early 1970s, at the start of the 1990s the consensus view in international finance was that PPP was of little use empirically and that as a result models of exchange rate behavior that used PPP as a building block, such as the monetary approach to the exchange rate (Frenkel and Johnson, 1978), were crucially flawed. Since that time, however, such assessments have had to be tempered as studies using both long historical time series (e.g. Frankel, 1986, 1990; Lothian and Taylor, 1996) or multi-country data for the floating exchange rate period alone (e.g. Flood and Taylor, 1996; Frankel and Rose, 1996; Lothian, 1997) or both (Taylor, 2002) have produced evidence supporting PPP at least as a long-run equilibrium relationship.

Among the questions that are currently central to this research agenda, therefore, are whether the results of such studies are robust and, relatedly, whether and to what extent other factors omitted from the simple equations used to test PPP might have mattered for real (and nominal) exchange rate behavior and hence may have been sources of bias. The purpose of this paper is to investigate some of these issues. To do so, we use an updated and otherwise revised version of the long-span historical data used in our earlier paper (Lothian and Taylor, 1996). These are annual data are for three countries, France, the United Kingdom and the United States, over a sample period that runs from 1820 to 2001.¹

Two sets of factors in particular have been cited as potential sources of problems. The first and perhaps most obvious is the omission of real variables affecting the equilibrium real rate: constancy of the equilibrium real exchange rate is implicit in PPP. Theory, however, suggests several important avenues through which real variables could affect the real exchange rate. The most prominent is through the effect of productivity differentials – the so-called Harrod-Balassa-Samuelson (HBS) effect.² Failure to account for the influence of such factors

¹While we have extended our data set from that used in Lothian and Taylor (1996) to include an additional ten years or so of data up to 2001, we have had to discard some observations at the beginning of the sample because the population data we use begins only in 1820. Nevertheless, the data set still spans over 180 years.

²Harrod (1933), Balassa (1964), Samuelson (1964).

could quite possibly be an important source of omitted-variable bias. One piece of evidence that such bias may be operating is the exceedingly slow estimated speeds of adjustment of real exchange rates back towards their mean values following shocks: the estimated half-lives of shocks typically range from three to five years. These seem much too long for adjustments to equilibrium – given that the short-run volatility of real exchange rates implies that they must be driven in the short-run largely by nominal shocks – so much so that Rogoff (1996) has termed this set of findings the ‘PPP puzzle’. Other researchers (e.g. Taylor, Peel and Sarno, 2001), in contrast, have hypothesized that these slow estimated speeds are due mainly to nonlinearities; due to factors such as transactions costs in international goods arbitrage, adjustments to large deviations from (measured) equilibrium are made quickly while small deviations can be much more persistent.

A third factor that may be pertinent when investigating the behavior of the real exchange rate over long spans of data is that there is considerable evidence that its volatility will typically shift according to the nominal exchange rate regime. In particular, there is well-documented evidence of considerably greater real exchange rate variability under floating rate regimes than under fixed rates (Frankel and Rose, 1995; Taylor, 1995).

We investigate all three factors below – HBS effects, regime-specific volatility effects and nonlinearities in adjustment.

Before turning to that evidence we review the empirical literature on PPP briefly in Section 2 of the paper in order to explain more fully the motivation for our research. We then go on in Sections 3 through 5 to discuss theoretical and empirical issues surrounding the questions under investigation. In Section 6 we outline the empirical procedures to be used and in section 7 describe the data. In Section 8 we present the empirical results themselves. In Section 9 we offer some brief conclusions.

2 The Existing Literature: A Brief Review

One way the empirical literature has sought to examine the validity of PPP as a long-run equilibrium condition has been to see whether the real exchange rate tends to settle down at a long-run equilibrium level – i.e. to see whether time series on real exchange rates appear to have been generated by ‘mean-reverting’ processes. While the property of mean reversion does not guarantee that the mean towards which the real exchange rate mean reverts is the PPP level, mean reversion is at least a necessary condition for long-run PPP to hold. As noted by Lothian and Taylor (1996), however, professional academic opinion on the validity of PPP itself seems to display mean reversion.

Prior to the 1970s, post-war academic opinion seemed to assume some form of *long-run* PPP, as evidenced, for example, by the classic study of Friedman and Schwartz (1963). Then, with the rise in dominance of the monetary approach to the exchange rate concomitant with the switch to floating exchange rates among the major industrialized countries in the early 1970s, there seemed

to be a move towards belief in *continuous* PPP (see, for example, the studies in Frenkel and Johnson, 1978). However, the poor empirical performance of monetary models of the exchange rate, as experience with floating rates unfolded, together with less formal but nevertheless compelling evidence of the excess volatility of nominal exchange rates compared to movements in relative national price levels, led to an acknowledgment of the ‘collapse of PPP’ by even its erstwhile foremost advocates by the end of the 1980s (Frenkel, 1981). While Dornbusch’s ‘overshooting’ refinement of the Mundell-Fleming model, by providing a rationale for short-run deviations from PPP (Dornbusch, 1976), did somewhat reinstate the professional standing of the concept, empirical evidence published mostly in the 1980s, appeared to drive the final nails in the coffin of PPP, even considered as a condition that should hold only on average and over long periods of time.³ In particular, neither Roll (1979) nor Adler and Lehmann (1983) could reject the null hypothesis of random-walk behavior in deviations from PPP (the real exchange rate) and subsequent cointegration studies similarly found no evidence of long-run PPP (Taylor, 1988; Mark, 1990). Thus, the professional consensus shifted yet again to a position opposite to the strongly held belief in continuous PPP which had held sway barely a decade before – i.e. towards a view that PPP was of virtually no use empirically over any time horizon (e.g. Stockman, 1987).⁴

Following Frankel (1986), however, a number of authors noted that the tests typically employed during the 1980s to test for long-run stability of the real exchange rate may have very low power to reject a null hypothesis of real exchange rate instability when applied to data for the recent floating rate period alone (e.g. Froot and Rogoff, 1995; Lothian and Taylor, 1996, 1997; Sarno and Taylor, 2002a). The argument is that if the real exchange rate is in fact stable in the sense that it tends to revert towards its mean over long periods of time, then examination of just one real exchange rate over a period of twenty years or so may not yield enough information to be able to detect slow mean reversion towards purchasing power parity. This led to two developments in research, both aimed at circumventing the problem of low power displayed by conventional unit root tests applied to the real exchange rate. In the first of these developments, researchers sought to increase the power of unit root tests by increasing the length of the sample period under consideration (e.g. Frankel, 1986; Diebold, Husted and Rush, 1991; Cheung and Lai, 1993a; Lothian and Taylor, 1996). These studies have in fact been able to find significant evidence of real exchange rate mean reversion. In the second line of development of this research, researchers sought to increase test power by using panel unit root tests applied jointly to a number of real exchange rate series over the recent float and, in many of these studies, the unit-root hypothesis is also rejected for groups of real exchange rates (e.g. Abuaf and Jorion, 1990; Frankel and Rose, 1996; Wu,

³This increasingly drew on the new literature on unit roots and cointegration introduced through the circulation and publication of Engle and Granger’s (1987) seminal paper.

⁴Note, however, that long-run PPP still appeared to explain real exchange rate behavior during various historical periods – in particular the interwar period (Taylor and McMahon, 1988).

1996; Flood and Taylor, 1996; Taylor and Sarno, 1998)⁵.

As with pooling more generally, a potential problem with these studies is the joint nature of the hypotheses being tested, in this instance that *all* of the series are generated by unit-root processes. Taylor and Sarno (1998), who first noted this problem, show that the problem of rejection of the joint null may be quite acute when as few as just one under consideration is a realization of a stationary process while the remainder are realizations of unit root processes. Often, however, researchers applying panel unit root tests have ignored this issue and, therefore, implicitly interpreted rejection of the joint null as indicative of stationarity of all of the series.⁶

Nevertheless, Rogoff (1996) notes that, even if we were to take the results of the long-span or panel-data studies as having provided evidence of significant mean reversion in the real exchange rate, these studies typically point to a half-life of deviations from PPP in the range of three to five years. If we take as given that real shocks cannot account for the major part of the short-run volatility of real exchange rates (since it seems incredible that shocks to real factors such as tastes and technology could be so volatile) and that nominal shocks can only have strong effects over a time frame in which nominal wages and prices are sticky, then the apparently high degree of persistence in the real exchange rate becomes something of a puzzle.

Taylor, Peel and Sarno (2001) argue that the key both to detecting significant mean reversion in the real exchange rate and to solving Rogoff's PPP puzzle lies in allowing for nonlinearities in real exchange rate adjustment. We discuss the rationale for nonlinear real exchange rate adjustment more fully in Section 3, although the intuition seems reasonably clear: the further the real exchange rate is from its long-run equilibrium, the stronger will be the forces driving it back towards equilibrium. The cause may be greater goods arbitrage as the misalignment grows (Obstfeld and Taylor, 1997) or a growing degree of consensus concerning the appropriate or likely direction of movements in the nominal exchange rate among traders (Kilian and Taylor, 2003), or perhaps a reter likelihood of the occurrence and success of intervention by the authorities to correct a misaligned exchange rate (Sarno and Taylor, 2001; Taylor, 2004). We discuss possible sources of nonlinearity in real exchange rate adjustment more fully below.

Parallel to the recent literature on nonlinearities in real exchange rate adjustment, researchers have also stressed the importance of real shocks to the underlying equilibrium real exchange rate (e.g. Engel, 1999, 2000). As discussed in Section 3, the idea that productivity shocks may affect the equilibrium real exchange rate – the so-called Harrod-Balassa-Samuels effect – has a fairly

⁵A third method for increasing the power of unit root tests, by employing univariate tests based on generalized or weighted least squares estimators, has recently been proposed by Cheung and Lai (1998).

⁶A number of studies of PPP behavior that use multi-country panel data as their units of observation have managed to avoid these problems. Taylor (2002), for example, examines such behavior for a group of twenty countries over a one hundred year period, thus combining the long-span and panel-data approaches, but takes care to avoid the panel-unit-root pitfall highlighted by Taylor and Sarno (1998).

long history in economics (Harrod, 1933; Balassa, 1964; Samuelson, 1964), although empirical evidence in its support appears to be relatively weak (Froot and Rogoff, 1995). Recently, however, Lothian and Taylor (2000) suggest that Harrod-Balassa-Samuelson effects may also be important in shedding light on the PPP puzzle: by allowing for underlying shifts in the equilibrium dollar-sterling real exchange rate over the past two hundred years through the use of nonlinear time trends, they find the estimated half-life of real exchange rate shocks substantially reduced even without explicit allowance for nonlinear real exchange rate adjustment. These results are similar to those reported by Lothian (1991) for yen-dollar and yen-sterling real exchange rates over the period 1875-1989. Allowing for a linear trend in the logarithm of real exchange rates results in as much as a 20% reduction (depending upon the exact specification of the model) in the half-lives of adjustment in both instances. As we point out below, failure to account for such effects may also be a reason for the better performance of models that incorporate nonlinearities.

3 Nonlinear Real Exchange Rate Adjustment

In this section we examine more fully the issue of nonlinearity in real exchange rate adjustment. The real exchange rate, q_t , may be expressed in logarithmic form as

$$q_t \equiv s_t + p_t - p_t^* \tag{1}$$

where s_t is the logarithm of the nominal exchange rate (foreign price of home currency), and p_t and p_t^* denote the logarithms of the home and foreign price levels respectively, all at time t . The real exchange rate may thus be interpreted as a measure of the deviation from PPP.

As noted above, it is now universally accepted that PPP certainly does not hold for major exchange rates *continuously*. A necessary condition for PPP to hold in a long-run sense, however, is that q_t should tend to settle down to an equilibrium level in the long-run. In statistical terms, this implies that the process generating q_t should be stationary. Given this and the casual empirical observation that (except during exceptional periods such as during a hyperinflation) major real exchange rates do not appear to be explosive, researchers have generally tested for long run PPP by testing the null hypothesis that the process generating the real exchange rate series is linear and borderline non-stationary in the sense that it has a unit root, while the alternative hypothesis is that all of the roots of the process lie within the unit circle. Thus, the maintained hypothesis in the conventional framework effectively assumes a linear autoregressive representation for the real exchange rate, which means that adjustment is both continuous and of constant speed, regardless of the size of the deviation from PPP.

Recently, however, a number of authors have suggested that adjustment in the real exchange rate towards the equilibrium may in fact be nonlinear. The possible sources of nonlinearity in real exchange rate adjustment have been discussed extensively in this literature (e.g. Taylor and Peel, 2000; Taylor, Peel

and Sarno, 2001; Sarno and Taylor, 2002a, 2002b; Kilian and Taylor, 2003; Taylor, 2003), so our treatment here will be brief.

One potential source of nonlinear adjustment in real exchange rates, which has been discussed in the literature as early as the work of Heckscher (1916), arises from nonlinearities in international goods arbitrage due to factors such as transport costs, tariffs and nontariff barriers which drive a wedge between the prices of similar goods in spatially separated markets (see, for example, Benninga and Protopapadakis, 1988; Dumas, 1992; Sercu, Uppal and Van Hulle, 1995). The basic framework in such models is straightforward. At a disaggregated level, proportional arbitrage costs create a band for the relative price of individual traded goods within which the marginal cost of arbitrage exceeds the marginal benefit. Hence, when the relative prices and the exchange rate is such that the deviation from the law of one price (LOP) for that good is small, the deviation is not worth arbitraging. Only when the deviation breaches a threshold determined by arbitrage costs does arbitrage take place. Aggregating these threshold effects across a basket of goods with varying levels of arbitrage costs means that the aggregate real exchange rate will tend to be indeterminate when it is near the PPP equilibrium and the LOP holds for most goods, and will become increasingly mean reverting back towards the PPP level as it diverges away from PPP and more and more goods breach the arbitrage costs thresholds and are arbitrated. Obstfeld and Taylor (1997) and Sarno, Taylor and Chowdhury (2004) provide empirical evidence of these kinds of threshold effects in deviations from the LOP, using disaggregated goods price data.

Kilian and Taylor (2003) suggest that another source of nonlinearity in the real exchange rate may be the heterogeneity of beliefs among market participants concerning the appropriate equilibrium level of the real exchange rate. They argue that the range of beliefs concerning the appropriate likely direction of change in the exchange rate in order to mean revert toward equilibrium will tend to narrow as the real exchange rate becomes increasingly misaligned with respect to the true but unobserved equilibrium level, since the exchange rate will tend to be on the same side of whatever equilibrium is postulated by individual agents for more extreme misalignments. As a consensus develops that a currency is overvalued or undervalued, therefore, market forces returning the exchange towards equilibrium will grow in strength as the deviation from equilibrium grows.

Taylor (2004) suggests and provides some empirical support for the idea that, during periods of managed floating, exchange rate intervention operations by the authorities are also likely to impart nonlinearities into the nominal and hence the real exchange rate, since intervention to correct misalignment is both more likely to occur and more likely to be effective the greater is the degree of misalignment. He argues that the effectiveness of intervention is likely to grow with the degree of misalignment on the basis that with larger misalignments, there is likely to be a greater number of market participants who will recognise and act on the authorities' coordinating signal to correct the misalignment.

Using data for the recent float alone, Taylor, Peel and Sarno (2001) record empirical results that provide strong confirmation that four major real bilat-

eral dollar exchange rates are well characterized by nonlinearly mean reverting processes over the floating rate period since 1973. They estimate smooth transition autoregressive (STAR) models – which we discuss below – for dollar-mark, dollar sterling, dollar-yen and dollar-franc. Their estimated models in each case imply an equilibrium level of the real exchange rate in the neighborhood of which the behavior of the log-level of the real exchange rate is close to a random walk, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium. Further, because of the nonlinear nature of the estimated models, the half-lives of shocks to the real exchange rates vary both with the size of the shock and with the initial conditions. Taylor, Peel and Sarno (2001) find that it is only for small shocks occurring when the real exchange rate is near its equilibrium that the nonlinear models consistently yield half lives in the range of three to five years. For dollar-mark and dollar-sterling in particular, even small shocks of one to five percent have a half life under three years. For larger shocks, the speed of mean reversion is even faster. These results therefore seem to go some way towards solving Rogoff’s (1996) PPP puzzle. In a number of Monte Carlo studies calibrated on their estimated nonlinear models, Taylor, Peel and Sarno (2001) also demonstrate the very low power of standard univariate unit root tests to reject a false null hypothesis of unit root behavior when the true model is nonlinearly mean reverting, thereby suggesting an explanation for the difficulty researchers have encountered in rejecting the linear unit root hypothesis at conventional significance levels for major real exchange rates over the recent floating rate period.

4 The Harrod-Balassa-Samuelson Effect

The key features of the Harrod-Balassa-Samuelson framework (Harrod, 1933; Balassa, 1964; Samuelson, 1964) may be set out informally as follows. Suppose a country experiences productivity growth primarily in its traded goods sector. Suppose also that the law of one price (LOP) holds among traded goods and, for ease of exposition though without loss of generality, assume that the nominal exchange rate remains constant. Productivity growth in the traded goods sector will lead to wage rises in that sector without the necessity for price rises. Hence traded goods prices can remain constant and the LOP can continue to hold with the unchanged nominal exchange rate. But workers in the non-traded goods sector will also demand comparable pay rises, and this will lead to a rise in the price of nontradables and hence an overall rise in the overall price index. Since the LOP holds among traded goods and, by assumption, the nominal exchange rate has remained constant, this means that the upward movement in the domestic price index will not be matched by a movement in the nominal exchange rate so that, if PPP initially held, the domestic currency must now appear overvalued on the basis of comparison made using price indices expressed in a common currency at the prevailing exchange rate. The crucial assumption is that productivity growth is much higher in the traded goods sector. Note also that the relative price of nontradables may rise even in the case of balanced

growth of the two sectors of the economy, as long as the nontraded goods sector is more labour intensive relative to the traded goods sector.

We can analyse this issue more formally using a fairly standard small open-economy model (Froot and Rogoff, 1991). Suppose that the home economy has two sectors, one producing tradeables and one producing nontradeables, each with a Cobb-Douglas technology of the form:

$$Y_I = A_I K_I^{\theta_I} \quad I = T, N \quad (2)$$

where Y_I , K_I and A_I denote, respectively, the output-labor ratio, the capital-labor ratio and productivity in sector I ($= T, N$) of the home economy, and the subscripts T and N denote the traded and nontraded sector respectively. We omit time subscripts for clarity. We assume long-run perfect factor mobility across sectors and long-run perfect competition in both traded and nontraded sectors. Thus, the long-run real rate of return to capital and the long-run real wage must be the same across sectors and equal to the relevant marginal product:

$$R = \theta_T A_T K_T^{\theta_T - 1} \quad (3)$$

$$R = \Pi_N \theta_N A_N K_N^{\theta_N - 1} \quad (4)$$

$$W = (1 - \theta_T) A_T K_T^{\theta_T} \quad (5)$$

$$W = \Pi_N (1 - \theta_N) A_N K_N^{\theta_N}, \quad (6)$$

where R denotes the real long-run return to capital and W is the long-run real wage, each measured in tradables, and Π_N is the relative price of nontradables. Taking logarithms (indicating logarithms by the use of lower case letters) and totally differentiating (3)-(6), we have:

$$da_T + (\theta_T - 1)dk_T = 0 \quad (7)$$

$$d\pi_N + da_N + (\theta_N - 1)dk_N = 0 \quad (8)$$

$$dw = da_T + \theta_T dk_T \quad (9)$$

$$dw = d\pi_N + da_N + \theta_N dk_N. \quad (10)$$

From which we derive:

$$dk_N = dk_T = dw = da_T (1 - \theta_T)^{-1} \quad (11)$$

and, in particular:

$$d\pi_N = (1 - \theta_N)(1 - \theta_T)^{-1} da_T - da_N. \quad (12)$$

Equation (12) incorporates the Harrod-Balassa-Samuelson condition that relatively higher productivity growth in the tradeables sector will tend to generate a rise in the relative price of nontradeables. The percentage change in the relative price of nontradables is, moreover, determined only by the production side of the economy, while demand factors do not affect the real exchange rate

in the long run. If the degree of capital intensity is the same across the traded and nontraded sectors, i.e. $\theta_T = \theta_N$, then the percentage change in relative prices is exactly equal to the productivity differential between the two sectors. Moreover, if the nontraded sector is less capital intensive than the traded sector, i.e. $\theta_N < \theta_T$, then even in a situation of balanced productivity growth in the two sectors ($da_T = da_N$), the relative price of nontradables will rise.

Integrating (12), we can derive an expression for the logarithm of the price of nontradables, p_N , relative to the price of tradables, p_T :⁷

$$p_N - p_T = (1 - \theta_N)(1 - \theta_T)^{-1} a_T - a_N. \quad (13)$$

Now suppose that the overall home price level, p , is a geometric weighted average of the home price of tradables and nontradables

$$p = \gamma p_T + (1 - \gamma) p_N, \quad (14)$$

or

$$p = p_T + (1 - \gamma)(p_N - p_T). \quad (15)$$

We assume that the law of one price holds among tradable goods:

$$p_T^* = p_T + s, \quad (16)$$

where an asterisk always denotes a foreign variable or a foreign coefficient. If we assume, moreover, that equations similar to (15) and (16) hold in the foreign economy, but perhaps with different coefficients, then from these three equations we can derive an expression for the equilibrium or long-run real exchange rate:

$$\begin{aligned} s - p^* + p &= (1 - \gamma) \left[(1 - \theta_N)(1 - \theta_T)^{-1} a_T - a_N \right] \\ &\quad - (1 - \gamma^*) \left[(1 - \theta_N^*)(1 - \theta_T^*)^{-1} a_T^* - a_N^* \right]. \end{aligned} \quad (17)$$

Suppose, further, that productivity in the nontradeables sector in each country is close to zero. In that case (17) collapses to

$$\begin{aligned} s - p^* + p &= (1 - \gamma) (1 - \theta_N)(1 - \theta_T)^{-1} a_T \\ &\quad - (1 - \gamma^*) (1 - \theta_N^*)(1 - \theta_T^*)^{-1} a_T^* \end{aligned} \quad (18)$$

or

$$q = \psi a_T - \psi^* a_T^*, \quad (19)$$

where q is the long-run equilibrium real exchange rate, $(s - p + p^*)$, $\psi = (1 - \gamma)(1 - \theta_N)(1 - \theta_T)^{-1} > 0$ and $\psi^* = (1 - \gamma^*)(1 - \theta_N^*)(1 - \theta_T^*)^{-1} > 0$.

⁷We ignore the constant of integration for simplicity, since it can always be removed from the solution by an arbitrary renormalization of variables.

Equation (19) expresses the essence of the Harrod-Balassa-Samuelson effect in its most succinct form: countries with relatively high levels of productivity will tend to have an uncompetitive equilibrium real exchange rate. Equivalently, rich countries will tend to have a higher exchange rate-adjusted price level on average.

The empirical evidence on the Harrod-Balassa-Samuelson effect is surveyed in Froot and Rogoff (1995). In general, the empirical evidence provides mixed results, with a preponderance of more recent studies finding at most very weak evidence of the Harrod-Balassa-Samuelson effect. For example, Froot and Rogoff (1991a,b), using cross-section data on 22 OECD countries over the period 1950-1989, find that the correlation between productivity levels and the real exchange rate is extremely weak. Asea and Mendoza (1994), using a data base including traded goods prices relative to nontraded goods prices for fourteen OECD countries over the period 1975-1985, find that although sectoral productivity differentials within countries do significantly explain relative nontraded goods prices *within* countries,⁸ changes in relative nontraded goods prices account for only a small and insignificant portion of real exchange rate changes across countries. In other words, they find little evidence of a Harrod-Balassa-Samuelson effect. As noted by Bergin, Glick and Taylor (2003), however, it may be that this effect has been variable over time, perhaps due to variations in relative productivity differentials, or other factors, and Lothian and Taylor (2000) conjecture that the presence of statistically significant cubic time trends in the real sterling-dollar series may be picking up HBS effects – an issue to which we return below.

5 The Volatility of the Real Exchange Rate Across Nominal Regimes

As documented by Frankel and Rose (1995), there is an abundance of empirical evidence which convincingly argues that the volatility of real exchange rates tends to vary across nominal exchange rate regimes and, in particular, tends to be much higher during floating-rate regimes. Influential studies which have reached this conclusion include Mussa (1986, 1990), Eichengreen (1988), Baxter and Stockman (1989) and Flood and Rose (1995). The Baxter and Stockman (1989) and Flood and Rose (1995) studies are particularly interesting in that they demonstrate that, although both real and nominal exchange rates tend to be much more volatile during floating exchange rate regimes, the underlying macro fundamental variables display no such regime-specific shifts in volatility.

This suggests that if one wishes to estimate a real exchange rate model spanning a number of nominal exchange rate regimes, it is important to allow for shifts in volatility in the error term of the empirical model. In their long-span real exchange rate study, Lothian and Taylor (1996) explicitly acknowledge this issue and allow for shifts in volatility in a very general way by using

⁸This is sometimes called the Baumol-Bowen effect (Baumol and Bowen, 1966).

heteroskedastic-robust estimation methods. In the present study, however, we specifically build in the possibility of shifts in volatility across nominal exchange rate regimes in designing our econometric model. Moreover, following the recent work of Reinhart and Rogoff (2004), which suggests that the actual behavior of exchange rates may not accord exactly with the officially recorded dates of exchange rate regimes, we allow for regime shifts in a flexible, data-instigated fashion, as described below.

6 Modelling Nonlinear Adjustment, Switching Volatility and The Harrod-Balassa-Samuelson Effect in the Real Exchange Rate

In the light of the discussion in the previous sections, it appears that nonlinearities in adjustment and shifts in volatility across exchange rate regimes are important features which should be allowed for in modelling the real exchange rate. In this section, we therefore describe our econometric procedures for allowing for these features and at the same time testing for statistically and economically significant Harrod-Balassa-Samuelson effects in an empirical real exchange rate model.

6.1 Modelling nonlinearity

One particular statistical characterization of nonlinear adjustment, which appears to work well for exchange rates, is the smooth transition autoregressive (STAR) model (Granger and Teräsvirta, 1993).⁹ In the STAR model, adjustment takes place in every period but the speed of adjustment varies with the extent of the deviation from parity. A STAR model for a process $\{q_t\}$ may be written:

$$[q_t - \mu_t] = \sum_{j=1}^p \beta_j [q_{t-j} - \mu_t] + \left[\sum_{j=1}^p \beta_j^* [q_{t-j} - \mu_{t-j}] \right] \Phi[\theta^2; q_{t-d} - \mu_{t-d}] + \varepsilon_t \quad (20)$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$ and $\theta^2 \in (0, +\infty)$. The transition function $\Phi[\theta^2; q_{t-d} - \mu_t]$ determines the degree of mean reversion and is itself governed by the parameter θ , which effectively determines the speed of mean reversion, and $\{\mu_t\}$ denotes the equilibrium level of $\{q_t\}$. Usually, $\{\mu_t\}$ is assumed to be constant to a first approximation. However, we relax this assumption in the present analysis in order to allow for Harrod-Balassa-Samuelson effects; our empirical specification of $\{\mu_t\}$ is discussed further below. Further, we also allow for shifts in variance in the error term $\{\varepsilon_t\}$, rather than assuming homoscedasticity as in previous studies of nonlinearity in real exchange rate movements. As discussed above, this seems particularly appropriate since our data span a number of ex-

⁹See, e.g., Taylor and Peel (2000); Taylor, Peel and Sarno (2001); and Kilian and Taylor (2003).

change rate regimes. The empirical specification of the residual variance is also discussed below.

A simple transition function suggested by Granger and Teräsvirta (1993) is the exponential function:

$$\Phi[\theta; q_{t-d} - \mu_{t-d}] = 1 - \exp[-\theta^2[q_{t-d} - \mu_{t-d}]^2] \quad (21)$$

in which case (20) would be termed an exponential STAR or ESTAR model. The exponential transition function is bounded between zero and unity, $\Phi : (0, \infty) \rightarrow [0, 1]$, has the properties $\Phi[0] = 0$ and $\lim_{x \rightarrow \pm\infty} \Phi[x] = 1$, and is symmetrically inverse-bell shaped around zero. An alternative formulation of the transition function employs a logistic transition function and results in the logistic STAR or LSTAR model. Since this implies asymmetric real exchange rate adjustment, however, it appears to be less attractive in the present context.

The transition parameter θ^2 determines the speed of transition between the two extreme regimes, with lower absolute values of θ^2 implying slower transition. The inner regime corresponds to $q_{t-d} = \mu$, when $\Phi = 0$ and (20) becomes a linear AR(p) model:

$$[q_{t-d} - \mu_{t-d}] = \sum_{j=1}^p \beta_j [q_{t-j} - \mu_{t-j}] + \varepsilon_t. \quad (22)$$

The outer regime corresponds, for a given θ^2 , to $\lim_{[q_{t-d} - \mu_{t-d}] \rightarrow \pm\infty} \Phi[\theta; q_{t-d} - \mu_{t-d}] = 1$, where (2) becomes a different AR(p) model:

$$[q_{t-d} - \mu_{t-d}] = \sum_{j=1}^p (\beta_j + \beta_j^*) [q_{t-j} - \mu_{t-j}] + \varepsilon_t \quad (23)$$

with a correspondingly different speed of mean reversion so long as $\beta_j^* \neq 0$ for at least one value of j .

A feature of the STAR family is that deviations from the equilibrium level only generate increasing mean reversion with a delay – of d periods in fact in equation (20). Of course, all econometric models are only an approximation to vastly more sophisticated real-world data generating processes, so that this feature should not worry us unduly. Nevertheless, in the modeling of real exchange rate behavior, economic intuition suggests a presumption in favor of smaller values of the delay parameter d rather than larger values, in that it is hard to imagine why there should be very long lags before the real exchange rate begins to adjust in response to a shock, especially where one is using annual data. Given that our economic intuition suggests, in the present application, a presumption in favor of an ESTAR over an LSTAR formulation and a low value for d , we used the method advocated by Taylor, Peel and Sarno (2001) for selecting the appropriate nonlinear model as follows. This may be described as follows. The order of the autoregression, p , is chosen through inspection of the PACF, as suggested by Granger and Teräsvirta (1993). A p -th order ESTAR model is then estimated with d set equal to unity. Lagrange multiplier (LM) tests of the form suggested by Eitrheim and Teräsvirta (1996) are then applied to the residuals of the estimated equation to test the hypothesis of no remaining nonlinearity for a range of values of d greater than unity. If significant

remaining nonlinearity is detected in the sense that the estimated value of θ^2 is found to be significantly different from zero at, say, the five percent level, d is increased, the model is re-estimated and the LM tests are applied again. Once a model is arrived at for which the LM tests for no remaining nonlinearity are all insignificant at the chosen significance level, then the specification search ceases. The residuals of that estimated equation are then tested for no remaining nonlinearity of the LSTAR type, again using LM tests of the type described by Eitrheim and Teräsvirta (1996), as a test of specification. A potential problem with this procedure is that the autoregressive parameters in the ESTAR model are not identified under the null hypothesis $H_0 : \theta^2 = 0$ which may potentially affect the distribution of the corresponding test statistic (Davies, 1987; Hansen, 1996). In practice, however, this problem can be circumvented by using the parametric bootstrap to calculate the empirical marginal significance level of the test statistic under the null hypothesis.

6.2 Allowing for the Harrod-Balassa-Samuelson Effect

In our discussion of nonlinear modelling so far, we have alluded to the possibility that the long-run equilibrium level of the real exchange rate may be time varying by allowing the term μ_t to carry a time subscript. In Section 3, however, we showed that the Harrod-Balassa-Samuelson model suggests that the long-run equilibrium real exchange rate should depend on the productivity of the tradeables and nontradeables sectors in the home and foreign economies. Further, if we make the approximation that productivity growth in the nontradeables sector is zero, then equation (19) suggests that the equilibrium real exchange rate will vary over time in response to variations in tradeables sector productivity in the two economies. In fact, under this assumption, variations in productivity in the economy as a whole will depend largely upon variation in productivity in the tradeables sector. Ideally, therefore, one would like to obtain data on tradeables sector output and employment over the sample period in order to measure tradeables sector productivity. Strictly speaking, we should also want to know the tradeables sector capital-labour ratio since, from (2), we have (ignoring sector subscripts) $a = y - \theta k$. As Froot and Rogoff (1995, p. 1680) note, however, “data on capital inputs is notoriously unreliable”, and this is likely to be the case *a fortiori* over a sample period spanning two centuries, as in the present study. Moreover, one should perhaps not take the simple, stylized model set out in Section 3 *too* literally when confronting real-world data. Rather, we should take from it the central implications of the Harrod-Balassa-Samuelson model, namely that if a country experiences relatively higher productivity growth, then it will suffer an appreciation of its exchange rate in real terms.

In fact, given data availability over such a long period of time, we propose to measure the productivity term by measuring productivity as the ratio of total national output, real GDP, to total population, as in the classic studies of Balassa (1964) and Officer (1976a,b). Hence, $\{\mu_t\}$, the long-run equilibrium level of $\{q_t\}$ in the STAR model (20), is modelled as:

$$\mu_t = \mu_0 + \mu_1 a_t + \mu_2 a_t^*, \quad (24)$$

with a_t^* and a_t measured as the logarithm of the ratio of real GDP to population in the foreign and home economies at time t , respectively.

6.3 Allowing for shifts in volatility across regimes

We are particularly concerned that there may have been a downward shift in the volatility of real exchange rates during fixed nominal exchange rate regimes, such as the Bretton Woods and Gold Standard periods. As demonstrated by Reinhart and Rogoff (2004), however, it is important not simply to impose constraints according to official regime classifications but, rather, to use the data to determine *de facto* rather than *de jure* nominal exchange rate regimes. Therefore, we used a method which allows the data to determine both the periods of shift in variance and the amount of the shift. In particular, a variance function that allows for multiple shifts in variance may be written:¹⁰

$$\sigma_t^2 = \gamma_0 + \sum_{i=1}^S \gamma_i \{1.0 + \exp[(1/100)(t - \text{int}(c_{i,1}))(t - \text{int}(c_{i,2}))]\}^{-1} \quad (25)$$

where $\text{int}(x)$ denotes the integer part of the real number x and $\text{int}(c_{i,1}) \leq \text{int}(c_{i,2})$. This can also be thought of as a smooth transition model in the second moments, since it allows a smooth transition between variance regimes. A major advantage of this functional form in the present context is that the switch points, namely the $c_{i,j}$ parameters, can be jointly estimated along with the other parameters of the model, rather than being imposed *a priori*.

The suggested switching variance model therefore allows for S switches in variance through the S terms in the summation, which are each of the form:

$$\Gamma[\gamma_i, \alpha_i, c_{i,1}, c_{i,2}, t] = \gamma_i \{1.0 + \exp[(1/100)(t - c_{i,1})(t - c_{i,2})]\}^{-1}.$$

This function is symmetric around $(c_{i,1} + c_{i,2})/2$, and has a minimum value of zero, which it attains for values of t much less than or much greater than $c_{i,1}$ and $c_{i,2}$: $\lim_{(t-c_{i,1})(t-c_{i,2}) \rightarrow +\infty} \Gamma = 0$. At the points $t = c_{i,1}$ and $t = c_{i,2}$, $\Gamma = \gamma_i/2$, while for values of t between $c_{i,1}$ and $c_{i,2}$, $t \in (c_{i,1}, c_{i,2})$, $\gamma_i/2 < \Gamma \leq \gamma_i$. Hence, according as to whether γ_i is positive or negative, the variance will increase or decrease between the two reference points $c_{i,1}$ $c_{i,2}$.

Within the switching variance framework we are proposing, we assume normally distributed but heteroscedastic errors for the ESTAR model, $\varepsilon_t \sim N(0, \sigma_t^2)$, so that equations (20) and (25) may be estimated jointly by maximum likelihood. In practice we set the initial number of switches in variance, S in equation

¹⁰Although switching functions of this kind have not previously been applied to variance functions, this functional form was suggested by the switching functions suggested in the context of switching regression parameters by Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and Teräsvirta (1998).

(25), equal to two, and started the estimation procedure with initial values of $c_{1,1} = 1$, $c_{1,2} = T/2 = c_{2,1}$, and $c_{2,2} = T$, where T is the total sample size. If all of the $c_{i,j}$ and γ_i parameters were found to be significantly different from zero, indicating at least two significant switches, we then inserted a third switch function between each of the segments in turn to see if another switch could be detected; and so forth.

Note that this kind of switching-variance model is quite different from models which employ Markov-switching in the second moments (see, e.g., Engel and Kim, 1999). In the model we are proposing, once the reference points have been determined, volatility of the real exchange rate is higher or lower in the various regimes and the probability of switching between regimes is zero until the next reference point arrives. In a Markov-switching variance model, on the other hand, there is a non-zero probability of switching from one regime to another at each point in time. Since we wish to identify distinct periods of low volatility in the real exchange rate corresponding to *de facto* nominal exchange rate regimes, the kind of approach we are suggesting seems to be more appropriate in the present analysis.

7 Data

For nominal exchange rates and aggregate prices, we used the series from Lothian and Taylor (1996) updated with data from the International Financial Statistics (IFS) CD-ROM data base. For a full description of the earlier data and their sources see the appendix to Lothian and Taylor (1996).

The real income data and population data used in this paper were constructed using a variety of sources. Data for UK real income for the period prior to 1864 were derived from Clark (2001). Data for UK real income for the periods 1864-69, 1870-1994 and 1995-2001 came from Feinstein (1972), Maddison (1995) and the IFS, respectively. Data for US real income came from Officer (2002) for 1791-1869, from Maddison (1995) for 1870-1994 and from the IFS thereafter. Data for French real income came from Toutain (1997) for 1815-1870, from Maddison for (1870-1994) and from the IFS thereafter. Data for UK population for 1791-1800 came from the POPULSTAT (<http://www.library.uu.nl/wesp/populstat/populhome.html>), website for 1801-1980, from Mitchell (1988) and for the remaining period from the IFS. Data for US population came from Populstat for 1791- 1994 and from the IFS thereafter. Data for French population came from Mitchell (1998) for 1815-1869, from Maddison (1995) for 1870-1994 and from the IFS thereafter.

8 Empirical Results

8.1 Linear estimation results

As a preliminary examination of the data, we tested for the presence of unit roots in the processes generating the time series, under the maintained hypothesis of

linearity. Accordingly, following Perron (1988) and Lothian and Taylor (1996), we estimated equations of the form:

$$q_t = \kappa + \lambda\left(t - \frac{T}{2}\right) + \delta q_{t-1} + u_t \quad (26)$$

where T is the sample size and u_t is an error term. The following null hypotheses are then tested:

$$H_A : \delta = 1; H_B : (\kappa, \lambda, \delta) = (0, 0, 1); H_C : (\lambda, \delta) = (0, 1), \quad (27)$$

using either the standard t -statistics and F -statistics, τ_τ (although referred to the distributions calculated by Fuller, 1976 and Dickey and Fuller, 1981), or the the corresponding transformations of these statistics due to Phillips (1987) and Phillips and Perron (1988), $Z(\tau_\tau)$, $Z(\Phi_2)$ and $Z(\Phi_3)$.¹¹ The advantage of using the Phillips-Perron transforms is that they allow in a nonparametric fashion for the possibility of serially correlated and heterogeneously distributed error terms. If the unit root hypothesis cannot be rejected at this stage, then greater test power may be obtained by estimating the equation:

$$q_t = \kappa^* + \delta^* q_{t-1} + u_t^* \quad (28)$$

and testing the hypotheses:

$$H_D : \delta^* = 1; H_E : (\kappa^*, \delta^*) = (0, 1), \quad (29)$$

using the corresponding t -statistics and F -statistics, τ_μ and Φ_1 (again referred to the Dickey-Fuller distributions), or their Phillips-Perron transformations, $Z(\tau_\mu)$ and $Z(\Phi_1)$. In Table 1 we show the results of applying these tests to the sterling-dollar and sterling-franc real exchange rate data for the sample periods 1820-2001. In each case, consistent with the results of Lothian and Taylor (1996), although using data sampled over a slightly different period, we are able to reject the unit root hypothesis at the five percent level or lower.

We then proceeded to estimate autoregressive models for each of the real exchange rates, with a lag length of one year, as suggested by examination of the partial autocorrelation function for each of the real exchange rate series. The results are reported in Table 2 and they are qualitatively similar to those reported by Lothian and Taylor (1996). Given our previous discussion concerning the importance of data span, it is perhaps not surprising, however, that the measured persistence of the tow real exchange rates is slightly higher than that reported in our earlier work, where we used a slightly longer data set (1820-2001 for sterling dollar, as opposed to 1791-1990 in our earlier paper, for example). Nevertheless, the point estimate of the autoregressive coefficient of

¹¹Phillips and Perron (1988) and Schwert (1989) demonstrate that the Phillips-Perron non-parametric test statistics may be subject to distortion in the presence of moving-average components in the time series. Accordingly, as in Lothian and Taylor (1996), we therefore tested for the presence of moving-average components and could detect no statistically significant such effects in either of the real exchange rate series.

0.902 for sterling dollar is close to the point estimate of 0.887 of Lothian and Taylor (1996), and implies a half-life of adjustment of 6.78 years. Again in line with Lothian and Taylor (1996), the results for the sterling-franc imply a faster speed of adjustment, with a point estimate of the autoregressive coefficient of 0.831 and a corresponding half-life estimate of 3.75 years. In brief, therefore, the linear estimation results are noteworthy for two reasons, both of which serve to confirm previous findings reported in the literature. First, it is possible to reject the unit root hypothesis at standard significance levels using sufficiently long spans of data (Frankel, 1986; Lothian and Taylor, 1996, 1997). Second, although the unit root hypothesis can be rejected, the estimated half-lives of shocks to the real exchange rates involved are extremely slow – ranging from about 3.75 to 6.78 years. Given that the volatility of real exchange rates implies that they must be largely driven by nominal and financial shocks which one would expect to mean revert at a much faster rate, this evidence is confirmatory of Rogoff’s ‘purchasing power parity puzzle’ (Rogoff, 1996).

Note, however, that for sterling-franc there is significant evidence of autoregressive conditional heteroscedasticity (ARCH) in the estimated residuals. Although we have used heteroskedasticity-robust estimated standard errors, this does suggest that it may be fruitful to try and model this heteroskedasticity directly.

8.2 Nonlinear estimation results

8.2.1 parameter estimates and residual diagnostics

There was no evidence of serial correlation in the real exchange rate series beyond first-order and, as predicted, a delay of one year appeared to capture adequately the nonlinear dynamics of the ESTAR model. Further, the coefficient on foreign productivity, when estimated freely, was numerically close to and insignificantly different from being equal and opposite to that on domestic productivity, so that productivity was entered in relative terms. For both exchange rates, moreover, only two regimes were identified where the variance of the residual appeared to switch. In each case the switch was to a lower variance.

The empirical model estimated in our final estimations, the results of which are reported in Table 3, may thus be summarized in the following equations:

$$[q_t - \mu_0 - \mu_1(a_t - a_t^*)] = [q_{t-1} - \mu_0 - \mu_1(a_{t-1} - a_{t-1}^*)] \quad (30)$$

$$\times \exp[-\theta^2[q_{t-1} - \mu_0 - \mu_1(a_{t-1} - a_{t-1}^*)]^2] + \varepsilon_t,$$

$$\varepsilon_t \sim N(0, \sigma_t^2), \quad (31)$$

$$\sigma_t^2 = \gamma_0 + \gamma_1\{1.0 + \exp[(1/100)(t - \text{int}(c_{1,1}))(t - \text{int}(c_{1,2}))]\}^{-1} \quad (32)$$

$$+ \gamma_2\{1.0 + \exp[(1/100)(t - \text{int}(c_{2,1}))(t - \text{int}(c_{2,2}))]\}^{-1}.$$

In both cases, a good fit is indicated, with the coefficient of determination in each case improving upon that obtained using a linear model. Moreover, the residual diagnostics (calculated using the residuals standardized by the square root of the estimated variance function) is in each case satisfactory. In each case, μ_0 was found to be insignificant at the five percent level and was set to zero. For both exchange rates, the unrestricted estimated integer parts of $c_{2,1}$ and $c_{2,2}$ were identical, and so the restriction $c_{2,1} = c_{2,2}$ was imposed. Also for sterling-franc, the coefficient μ_1 was found to be insignificant at the five percent level and was set to zero.

The estimation results are noteworthy for a number of reasons. First, there is significant evidence of nonlinear mean reversion, as shown by the fact that the estimated transition parameter θ^2 is in every case strongly significantly different from zero. Note that the ratio of this estimated coefficient to its standard error – the ‘*t*-ratio’ – cannot be referred to the Student-*t* or normal distribution for purposes of inference, because under the null hypothesis $H_0 : \theta^2 = 0$, q_t follows a unit root process. This introduces a singularity under the null hypothesis so that standard inference procedures cannot be used, analogously to the way in which standard inference procedures cannot be used in the usual Dickey-Fuller or augmented Dickey-Fuller tests for a linear unit root. Indeed, testing the null hypothesis $H_0 : \theta^2 = 0$ is tantamount to a test of the null hypothesis against the alternative hypothesis of nonlinear mean reversion, rather than against the alternative of linear mean reversion. Therefore, because the distribution of θ^2 is unknown under the null hypothesis, we calculated the empirical marginal significance level of the ratio of the estimated coefficient to the estimated standard error by Monte Carlo methods under the null hypothesis that the true data generating process for the logarithm of both of the real exchange rate series was a random walk, with the parameters of the data generating process calibrated using the actual real exchange rate data over the sample period.¹² From these empirical marginal significance levels (reported in square brackets below the coefficient estimates in Table 3), we see that the estimated transition parameter is significantly different from zero with a marginal significance level of virtually zero in each case. Since these tests may be construed as nonlinear unit root tests, the results indicate strong evidence of nonlinear mean reversion for each of the real exchange rates examined over the sample period.

Second, the estimated coefficient for the relative productivity term, μ_1 is strongly significantly different from zero for the case of sterling-dollar (an asymptotic *t*-ratio of nearly eight) and is correctly signed according to the Harrod-Balassa-Samuelson effect: relatively higher US productivity generates a real appreciation of the equilibrium value of the dollar against the pound. For the case

¹²The empirical significance levels were based on 5,000 simulations of length 280, initialized at $q(1) = 0$, from which the first 100 data points were in each case discarded. At each replication a system of ESTAR equations identical in form to those reported in Table 3 was estimated. The percentage of replications for which a ‘*t*-ratio’ for the estimated transition parameters greater in absolute value than that reported in Table 3 was obtained was then taken as the empirical marginal significance level in each case.

of sterling-franc, however, there is no significant evidence of the HBS effect.¹³

Third, although only two low-variance regimes are indicated over the sample period, it is remarkable that these correspond closely to the classical Gold Standard and Bretton Woods periods. In Figures 1 and 2 we have graphed the actual real exchange rates and the estimated variance function, with the shaded portions showing the classical Gold Standard and Bretton Woods periods. Moreover, there is now no significant evidence of ARCH effects in the standardised residuals from either equation.

8.2.2 calculating the average speed of mean reversion

We proceeded to gain a measure of the mean-reverting properties of the estimated nonlinear models through calculation of their implied half-lives. Effectively, this involves comparing the impulse-response functions of the models with and without initial shocks. Thus, we examined the dynamic adjustment in response to shocks through impulse response functions which record the expected effect of a shock at time t on the system at time $t + j$. For a univariate linear model, the impulse response function is equivalent to a plot of the coefficients of the moving average representation (see e.g. Hamilton, 1994, p. 318). Estimating the impulse response function for a nonlinear model, however, raises special problems both of interpretation and of computation (Gallant, Rossi and Tauchen, 1993; Koop, Pesaran and Potter, 1996). In particular, with nonlinear models, the shape of the impulse-response function is not independent with respect to either the history of the system at the time the shock occurs, the size of the shock considered, or the distribution of future exogenous innovations. Exact estimates can only be produced – for a given shock size and initial condition – by multiple integration of the nonlinear function with respect to the distribution function each of the j future innovations, which is computationally impracticable for the long forecast horizons required in impulse response analysis.

In this paper, we calculate the impulse response functions, both conditional on average initial history and conditional on initial real exchange rate equilibrium, using the Monte Carlo integration method discussed by Gallant, Rossi and Tauchen (1993). The basic idea is to calculate a baseline forecast for a large number of periods ahead using the estimated model. We then calculate a second forecast but this time with a shock in the initial period. In both cases, the forecast is calculated by simulating the model a large number of times and taking the average of the various simulations as the forecast. The difference between the two forecasts is then taken as the impulse response function. The discrete number of years it takes for the impulse response function to drop below fifty percent is then taken as the estimated half life for that size of shock.

All simulations were carried out using initial values corresponding to the post-Bretton Woods period, 1973-2001. In our first estimation of the impulse response functions we condition on initial equilibrium by setting the initial

¹³These results are in line with the present authors' conjecture in Lothian and Taylor (2000), based on an analysis of nonlinear trends in these real exchange rates.

lagged values of the real exchange rate equal to the estimated equilibrium level, given the lagged value of relative productivity and the estimated coefficients: $\widehat{\mu}_1(a_{t-1} - a_{t-1}^*)$ (where $\widehat{\mu}_1$ denotes the fitted value of μ_1 and μ_0 was set to zero). We then used a total of 5,000 replications to produce each next-step-ahead forecast in the sequence, conditional on the previous forecast, and took the average over the 5,000 as the forecast value for that step. This is done for 100 steps ahead, with and without an additive shock at time t and the sequence representing the difference between the two paths is taken as the impulse response function. Since we use a large number of simulations, by the Law of Large Numbers this procedure should produce results virtually identical to that which would result from calculating the exact response functions analytically by multiple integration (Gallant, Rossi and Tauchen, 1993).

This procedure was then modified as follows in order to produce an estimate of the impulse-response function conditional on the average history of each of the real exchange rates. Starting at the first data point (for 1974), q_{t-1} is set equal to $\{|q(1973) - \widehat{\mu}_0 - \widehat{\mu}_1(a_{1973} - a_{1973}^*)| + (\widehat{\mu}_0 + \widehat{\mu}_1(a_{1973} - a_{1973}^*))\}$. If $[q(1973) - \widehat{\mu}_0 - \widehat{\mu}_1(a_{1973} - a_{1973}^*)]$ is positive, this is just $[q(1973) - \widehat{\mu}_0 - \widehat{\mu}_1(a_{1973} - a_{1973}^*)]$ itself. If, however, $[q(1973) - \widehat{\mu}_0 - \widehat{\mu}_1(a_{1973} - a_{1973}^*)]$ is negative, then $\{|q(1973) - \widehat{\mu}_0 - \widehat{\mu}_1(a_{1973} - a_{1973}^*)| + (\widehat{\mu}_0 + \widehat{\mu}_1(a_{1973} - a_{1973}^*))\}$ is the value which is an equal absolute distance above the estimated equilibrium value $\widehat{\mu}_0 + \widehat{\mu}_1(a_{1973} - a_{1973}^*)$. This transformation is necessary because we consider only positive shocks and it is innocuous because of the symmetric nature of ESTAR adjustment below and above equilibrium. A 100-step forecast is then produced using 200 replications at each step, with and without a positive shock of size s_t at time t , using the estimated ESTAR model, and realizations of the differences between the two forecasts are calculated and stored as before. We then move up one data point (hence setting $t - 1 = 1974$), and repeat this procedure. Once this has been done for every data point in the sample up to 2001, an average over all of the simulated sequences of differences in the paths of the real exchange rates with and without the shock at time t is taken as the estimated impulse response function conditional on the average history of the given exchange rate and for a given shock size.

For linear time series models the size of shock used to trace out an impulse response function is not of particular interest since it serves only as a scale factor, but it is of crucial importance in the nonlinear case. In the present application we are particularly concerned with the effect of shocks to the level of the real exchange rate. Given a particular value of the log real exchange rate at time t , q_t – whether this be the historical value or the estimated equilibrium level – a shock of k percent to the level of the real exchange rate involves augmenting q_t additively by $s_t = \log(1 + k/100)$.¹⁴ This raises a problem, however, in the calculation of the half-lives, since although the natural measure might be the discrete number of years taken until the shock to the level of the real exchange rate has dissipated by a half – i.e. when the impulse response function falls below

¹⁴For small k , $\log(1 + k/100)$ is of course approximately equal to $k/100$. This approximation is not, however, good for the larger shocks considered in this paper.

$\log(1 + k/200)$ – this would make comparisons with previous research on linear time series models of real exchange rates difficult. Accordingly, although we define a k percent shock to the real rate as equivalent to adding $\log(1 + k/100)$ to q_t , we calculate the half life as the discrete number of years taken for the impulse response function to fall below $0.5\log(1 + k/100)$, facilitating a comparison of our results with half lives estimated in previous studies. We considered six different sizes of percentage shock to the level of the real exchange rate, $k \in \{1, 5, 10, 20, 30, 40\}$. This allows us to compare and contrast the persistence of very large and very small shocks.

The estimated half-lives of the two real exchange rate models, calculated for the six sizes of shock, conditional on average initial history over the period 1973 – 2001, or on initial equilibrium, are shown in Table 4. They illustrate well the nonlinear nature of the estimated real exchange rate models, with larger shocks mean reverting much faster than smaller shocks and shocks conditional on average history mean reverting much faster than those conditional on initial equilibrium. In particular, for shocks of ten percent or less and conditional on average initial history, the half-life is in both cases two years, while larger shocks have a half life of one year or less. These results therefore accord broadly with those reported in Taylor, Peel and Sarno (2001), and shed some light on Rogoff’s (1996) ‘PPP puzzle’. Only for small shocks occurring when the real exchange rate is near its equilibrium do our nonlinear models consistently yield very long half lives in the range of three to five years or more, which Rogoff (1996) terms ‘glacial’. Once nonlinearity is allowed for, even small shocks of one to five percent have a half life of two years or less, conditional on average history, and for larger shocks the speed of mean reversion is even faster.

8.3 How important is the Harrod-Balassa-Samuelson Effect?

Our nonlinear estimation results indicate that the Harrod-Balassa-Samuelson effect is strongly statistically significant in explaining movements in the equilibrium real exchange rate for sterling-dollar but not for sterling-franc over the hundred-and-eighty-year period under investigation. However, statistical significance is not quite the same thing as economic significance. In particular, if the Harrod-Balassa-Samuelson effect has been *economically* significant, we should perhaps expect it to account for a substantial proportion of the variation in the real exchange rate over the period in question.

In Table 5, we report the results of some simple investigations of the importance of the HBS effect for sterling dollar. In panel a) we report the results of regressing the real exchange rate onto the relative productivity term alone, HBS_t , measured as $HBS_t = \widehat{\mu}_1(a_t - a_t^*)$, where $\widehat{\mu}_1$ is the estimated value of μ_1 from Table 3.¹⁵ The point estimate of the slope coefficient is close to unity, as one might expect, and is highly significant. The most striking feature of this

¹⁵Note that $\widehat{\mu}_1$ serves solely as a scale parameter in these regressions so that the fact that it is estimated has no bearing on any of the statistical inferences drawn from the results reported in Table 5, apart from the point estimate of the slope coefficient in panel a).

simple regression, however, is that the HBS effect appears to account for just over 40% of variation in the real exchange rate over the period in question. This accords with Rogoff's (1996) intuition that real exchange rate variation is driven largely by nominal shocks (some 60% on our measure), although a contribution of 40% is clearly sizeable.

Following our analysis in Lothian and Taylor (2000), in panel b) of Table 5 we have introduced a linear and a cubic trend into the first-order autoregression for the sterling-dollar real exchange rate and both are found to be significant and the effect is to reduce the point estimate of the autoregressive coefficient and hence the estimated half-life substantially. We conjectured in our earlier paper that these trends were in fact proxying HBS effects. In panel c) we have added the HBS term to the regression in panel b). The effect is to render the trend terms both individually and jointly insignificant at the 5% level or better, while the HBS term itself is strongly significant (with an asymptotic t -ratio of 2.571). This therefore appears to confirm our previous conjecture.

Finally, we report in panel d) of Table 5 the result of adding the HBS term to a simple autoregression for the sterling-dollar real exchange rate. The HBS term is again strongly significant (asymptotic t -ratio of 2.827), and the estimated half-life of adjustment drops from a value of 6.78 recorded when the HBS term is absent (Table 2) to a value of 3.75, a reduction of some 45%.

While the results are clearly only indicative, especially given the importance we have demonstrated of allowing for nonlinear adjustment in real exchange rates, they are nevertheless interesting for at least three reasons. First, they allow us to encompass results reported in previous work; secondly, they demonstrate that the HBS effect accounted for a sizeable proportion – some 40% – of the variation in the sterling-dollar real exchange rate over the period in question; thirdly, they suggest that nominal effects were in fact probably largely responsible for the remaining 60% of real exchange rate variation for sterling dollar

In Figure 3 we have plotted the sterling-dollar real exchange rate together with our measure of the Harrod-Belassa-Samuelson term, $HBS_t = \widehat{\mu}_1(a_t - a_t^*)$; it is striking how relative productivity is capturing the underlying trend depreciation of the real value of sterling against the dollar over this very long period.

In terms of the proportion of the total variation in the real exchange rate explained, therefore, it appears that, at least for sterling dollar, HBS effects are economically as well as statistically significant. However, a slightly different perspective may be obtained by comparing the variation in the real exchange rate and in relative productivity to the variation of national price levels expressed in a common currency. In Figure 4 we have plotted q_t and HBS_t again for sterling-dollar, but in addition we have also plotted on the same graph the logarithm of the UK price level in dollar terms i.e. $(s_t + p_t)$, and the logarithm of the US price level, p_t^* .

These graphs are interesting for at least two reasons.

First, while the first panel of Figure 4 appears to show quite a lot of variation in the equilibrium real exchange rate, the variation over the whole sample period

is dramatically dwarfed when compared to variation in the real exchange rate itself or to variations in national price levels.

Second, the close correspondence of $(s_t + p_t)$ and p_t^* over the whole sample period is very marked, and provides strong visual evidence that PPP appears to play a strong role in driving real exchange rates. The price levels in both instances have substantial and quite similar upward trends. The real exchange rates, in contrast, show no marked trend movement. The major implications of purchasing power parity are, therefore, visually confirmed: national price levels expressed in a common currency move together closely over the long term, while the real exchange rate over such time horizons appears highly stable in comparison to the nominal data.

9 Conclusions

With the rehabilitation of purchasing power parity during the past decade, researchers have turned to a new set of questions with regard to exchange-rate behavior. The key issues now are what else matters for nominal-exchange-rate, or alternatively, relative-price-level behavior and why the estimated speeds of adjustment of real exchange rates to their (measured) equilibrium values appear to be so slow.

These two questions, which as it turns out are more closely linked than may at first be apparent, have been the subject of investigation in this paper. In answering them, we have focused on three issues in particular. The first is the effect of real variables on the equilibrium levels of real exchange rates over the long run. Prominent in the theoretical literature, and the specific focus here, are differences in relative productivity growth – the Harrod-Balassa-Samuelson effect. A second issue, which has been highlighted in the recent empirical literature, is the possibility of nonlinear adjustment of real exchange rates to their long-run equilibria. A third influence stressed in the empirical literature is differences in real exchange rate volatility across nominal exchange rate regimes.

We have investigated all three sets of influences. To do so, we have estimated exponential smooth transition autoregressive (ESTAR) models for real dollar-sterling and real dollar-franc exchange rates in which we include relative real per capita income as a proxy for relative productivity and in which we allow for possible shifts in the variance of the errors. The data set that we use is a modified version of the data set used in our earlier (1996) study. Here the data begin in 1820 rather than 1791 as in our earlier study and end in 2002 rather than 1992. We omit the initial three decades because of difficulties in constructing real per capita income data for that period. The end result, therefore, is a sample 183 years in length. We use these data because they are well known, both as a result of our initial study and of the numerous subsequent studies that have re-examined and otherwise extended our earlier findings results (e.g. Cuddington and Liang, 2000; Hegwood and Papell, 1998) and because of their long span. The latter, as we have argued elsewhere (Lothian and Taylor, 1997), is necessary for adequate test power in both the usual statistical sense and the

broader economic sense of providing an environment in which the other factors that in principle can affect real exchange-rate behavior have sufficient scope to operate.

We find evidence of HBS effects for dollar-sterling but not for franc-sterling. These effects, moreover, are significant in an economic as well as statistical sense. The shifts in variance that we identify are chosen statistically rather than on the basis of a priori considerations. As it turns out, periods of low variance correspond almost perfectly to periods in which nominal exchange rates were rigidly or largely fixed (the gold-standard and Bretton-Woods eras, respectively), and periods of high variance to periods in which nominal exchange rates were subject to much greater fluctuation (principally the post-Bretton-Woods era). At the same time, we also find evidence of significant nonlinear mean reversion.

We then go on to analyze the impulse-response functions for shocks of varying magnitudes to the two real exchange rates. In both instances, these show greatly increased speeds of adjustment vis-à-vis those estimated with linear autoregressive models for all but the very smallest shocks. Conditional on average initial history, the estimated half lives for large shocks of twenty per cent or more are only one year; for small shocks in the range of one to five percent they range from one to two years depending upon the exact magnitude of the shocks.

What does all of this add up to for PPP? For the franc-sterling exchange rate, we found evidence of nonlinear mean reversion towards a constant equilibrium real exchange rate, which is strongly supportive of long-run purchasing power parity. For sterling-dollar, however, we found evidence of nonlinear reversion towards a shifting equilibrium real exchange rate that is a function of real productivity differentials, so that strict PPP is violated even as a long-run condition for this real exchange rate. US-UK Productivity differentials are, moreover, able to account for long-term trends in the real exchange rate and explain around 40% of its variation.¹⁶ To leave it at that would, however, be to perform Hamlet without the prince, as the graphical comparisons with which we conclude our analysis so clearly indicate. Consistent with the broader implications of PPP, the variations in the real exchange rate in each instance appear small when plotted on the same scale as the price level in the one country and the exchange-rate-adjusted price level in the other. The real exchange rate in each instance appears mean-reverting, while the common-currency price series, in contrast, have substantial upward trends. Those price-level trends, moreover, in both instances, appear virtually the same, again consistent with the broad implications of the PPP hypothesis, notwithstanding the presence of a statistically significant Harrod-Balassa-Samuelson effect for dollar-sterling.

¹⁶This evidence is consistent with Engel and Kim's (1999) study of the US-UK real exchange rate, in that those authors, using Kalman filtering techniques, find a persistent component of the real rate which appears to be highly correlated with productivity differentials.

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Table 1: Linear Unit Root Tests for Real Exchange Ratesa) *Sterling-Dollar 1820-2001*

τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
-3.19	-3.44	4.89	4.06	6.07
$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
-3.23	-3.69	5.23	4.62	6.91

b) *Sterling-Franc 1820-2001*

τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
-3.72	-3.73	6.96	4.92	7.32
$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
-3.86	-3.85	7.48	5.21	7.76

Note: The null hypotheses for each of the test statistics are given in the text and defined in Perron (1988). A Newey-West window of width 4 was used for the non-parametric corrections (Newey and West, 1987), although experiments with different band-widths led to little difference in the results. The asymptotic critical values for the statistics at various test sizes are as follows (Fuller, 1976; Dickey and Fuller, 1981):

	10%	5%	2.5%	1%
$\tau_\mu, Z(\tau_\mu)$	-2.57	-2.86	-3.12	-3.43
$\tau_\tau, Z(\tau_\tau)$	-3.12	-3.41	-3.66	-3.96
$\Phi_1, Z(\Phi_1)$	3.78	4.59	5.38	6.43
$\Phi_2, Z(\Phi_2)$	4.03	4.68	5.31	6.09
$\Phi_3, Z(\Phi_3)$	5.34	6.25	7.16	8.27

Table 2: Estimated Linear Autoregressions

a) *Sterling-Dollar 1820-2001*

$$\hat{q}_t = \begin{array}{r} -.007 \\ (-1.401) \end{array} + \begin{array}{r} 0.902 \\ (28.188) \end{array} q_{t-1}$$

$$\begin{aligned} R^2 &= 0.82; \quad SER = 6.45\%; \quad DW = 1.75; \\ AR(1) &= [0.08]; \quad ARCH(1) = [0.25]; \quad HL = 6.78. \end{aligned}$$

b) *Sterling-Franc 1820-2001*

$$\hat{q}_t = \begin{array}{r} -.009 \\ (-1.286) \end{array} + \begin{array}{r} .831 \\ (12.043) \end{array} q_{t-1}$$

$$\begin{aligned} R^2 &= .65; \quad SER = 7.5\%; \quad DW = 2.01; \\ AR(1) &= [0.85]; \quad ARCH(1) = [0.00]; \quad HL = 3.57. \end{aligned}$$

Note: Figures in parentheses below estimated coefficients are asymptotic t-ratios, calculated using heteroskedastic-consistent estimated standard errors (White, 1980); figures in square brackets are marginal significance levels. R^2 is the coefficient of determination, SER is the standard error of the regression, DW is the Durbin-Watson statistic, $AR(1)$ is a lagrange multiplier statistic for first-order serial correlation of the residuals, $ARCH(1)$ is lagrange multiplier statistic for first-order autoregressive heteroskedasticity in the residuals, and HL is the implied estimated half-life of real exchange rate shocks.

Table 3: Estimated Nonlinear Modelsa) *Sterling-Dollar 1820-2001*

μ_0	μ_1	θ^2	γ_0	γ_1
0.000	0.177	2.965	0.005	-0.004
(-)	(7.922)	(2.831)	(8.844)	(-5.567)
		[0.009]		
$c_{1,1}$	$c_{1,2}$	γ_2	$c_{2,1}$	$c_{2,2}$
49.741	89.781	-0.008	138.357	138.357
(7.430)	(18.416)	(-7.516)	(181.108)	(181.108)

$$\begin{aligned}
 R^2 &= .86; \quad SER = 6.0\%; \\
 AR(1) &= [0.11]; \quad ARCH(1) = [0.48]; \\
 NL - ESTAR &= [0.52]; \quad NL - LSTAR = [0.66].
 \end{aligned}$$

Implied low-variance reference dates: {1868, 1908}, {1957, 1957}

b) *Sterling-Franc 1820-2001*

μ_0	μ_1	θ^2	γ_0	γ_1
0.000	0.00	2.763	0.012	-0.011
(-)	(-)	(3.016)	(7.845)	(-7.548)
		[0.001]		
$c_{1,1}$	$c_{1,2}$	γ_2	$c_{2,1}$	$c_{2,2}$
45.975	96.061	-0.0232	141.664	141.664
(16.713)	(66.559)	(-6.480)	(179.201)	(179.201)

$$\begin{aligned}
 R^2 &= .71; \quad SER = 7.2\%; \\
 AR(1) &= [0.74]; \quad ARCH(1) = [0.83]; \quad HBS(\mu_1 = 0) = [0.15]; \\
 NL - ESTAR &= [0.67]; \quad NL - LSTAR = [0.77].
 \end{aligned}$$

Implied low-variance reference dates: {1864, 1915}, {1960, 1960}

Note: Figures in parentheses below estimated coefficients denote the ratio of the estimated coefficient to the estimated standard error (the asymptotic ‘t-ratio’);

figures in square brackets are marginal significance levels. The marginal significance level for the null hypothesis $H_0 : \theta^2 = 0$ was calculated by Monte Carlo methods, as described in the text. R^2 is the coefficient of determination, SER is the standard error of the regression, $AR(1)$ is a lagrange multiplier statistic for first-order serial correlation of the residuals and $ARCH(1)$ is lagrange multiplier statistic for first-order autoregressive heteroskedasticity in the residuals. $HBS(\mu_1 = 0)$ is a Wald test statistic for the parameter on relative productivity to be zero. $NL - ESTAR$ and $NL - LSTAR$ are lagrange mulitplier statistics for the hypothesis of no remaining nonlinearity of the ESTAR and LSTAR variety respectively.

Table 4: Estimated Half-Lives for the Nonlinear Models

a) *Conditional on average initial history*

Shock (%):	40	30	20	10	5	1
Dollar-Sterling	1	1	1	2	2	2
Dollar-Franc	1	1	1	2	2	2

b) *Conditional on initial exchange rate equilibrium*

Shock (%):	40	30	20	10	5	1
Dollar-Sterling	1	1	2	4	6	9
Dollar-Franc	1	1	2	3	4	6

Table 5: The Harrod-Balassa-Samuelson Effect and the Sterling-Dollar Exchange Rate

a) *Regression of real exchange rate onto HBS*

$$\hat{q}_t = \begin{array}{c} 0.933 \\ (10.750) \end{array} HBS_t$$

$$R^2 = 0.42; \quad SER = 12.92\%.$$

b) *Autoregression of real exchange rate with a cubic trend*

$$\hat{q}_t = \begin{array}{c} 0.030 \\ (2.115) \end{array} + \begin{array}{c} 0.826 \\ (19.195) \end{array} q_{t-1} - \begin{array}{c} 7.639 \times 10^{-4} \\ (-2.697) \end{array} \tau + \begin{array}{c} 1.750 \times 10^{-8} \\ (2.159) \end{array} \tau^3$$

$$R^2 = 0.83; \quad SER = 6.36\%; \quad DW = 1.68; \quad HL = 3.63.$$

c) *Autoregression of real exchange rate with a cubic trend and HBS*

$$\hat{q}_t = \begin{array}{c} .029 \\ (2.090) \end{array} + \begin{array}{c} 0.812 \\ (19.274) \end{array} q_{t-1} + \begin{array}{c} 2.718 \times 10^{-4} \\ (0.480) \end{array} \tau + \begin{array}{c} 6.447 \times 10^{-9} \\ (0.650) \end{array} \tau^3 + \begin{array}{c} 0.745 \\ (2.240) \end{array} HBS_t$$

$$R^2 = 0.84; \quad SER = 6.26\%; \quad DW = 1.68; \quad W(No Trends) = [0.08]; \quad HL = 3.32.$$

d) *Autoregression of real exchange rate with HBS*

$$\hat{q}_t = \begin{array}{c} .019 \\ (1.839) \end{array} + \begin{array}{c} 0.831 \\ (19.452) \end{array} q_{t-1} + \begin{array}{c} 0.314 \\ (2.732) \end{array} HBS_t$$

$$R^2 = 0.83; \quad SER = 6.33\%; \quad DW = 1.68; \quad HL = 3.75.$$

Note: Figures in parentheses below estimated coefficients are asymptotic t-ratios, calculated using heteroskedastic-consistent estimated standard errors (White, 1980); figures in square brackets are marginal significance levels. τ_t is a linear time trend, $\tau_{1820} = 1.0$, and HBS_t is the Harrod-Balassa-Samuelson effect: $HBS_t = \hat{\mu}_1(a_t - a_t^*)$, where $\hat{\mu}_1$ is the estimated value of μ_1 in Table 3. R^2 is the coefficient of determination, SER is the standard error of the regression, DW is the Durbin-Watson statistic, $W(No Trends)$ is a Wald test for the joint significance of the two trend parameters, and HL is the implied estimated half life of real exchange rate shocks.

Figure 1: Estimated Low-Variance Regimes for Dollar-Sterling

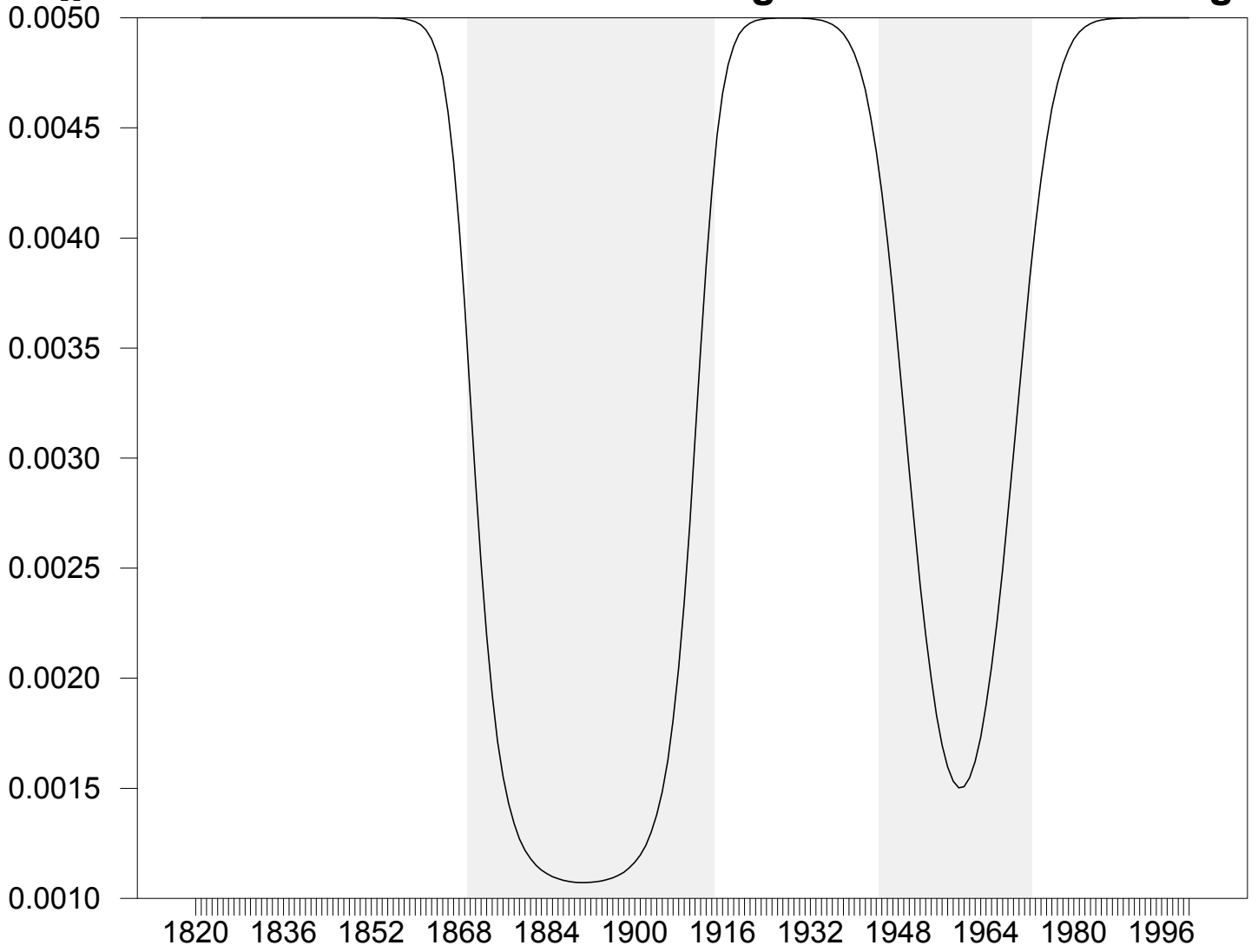


Figure 2: Estimated Low-Variance Regimes for Franc-Sterling

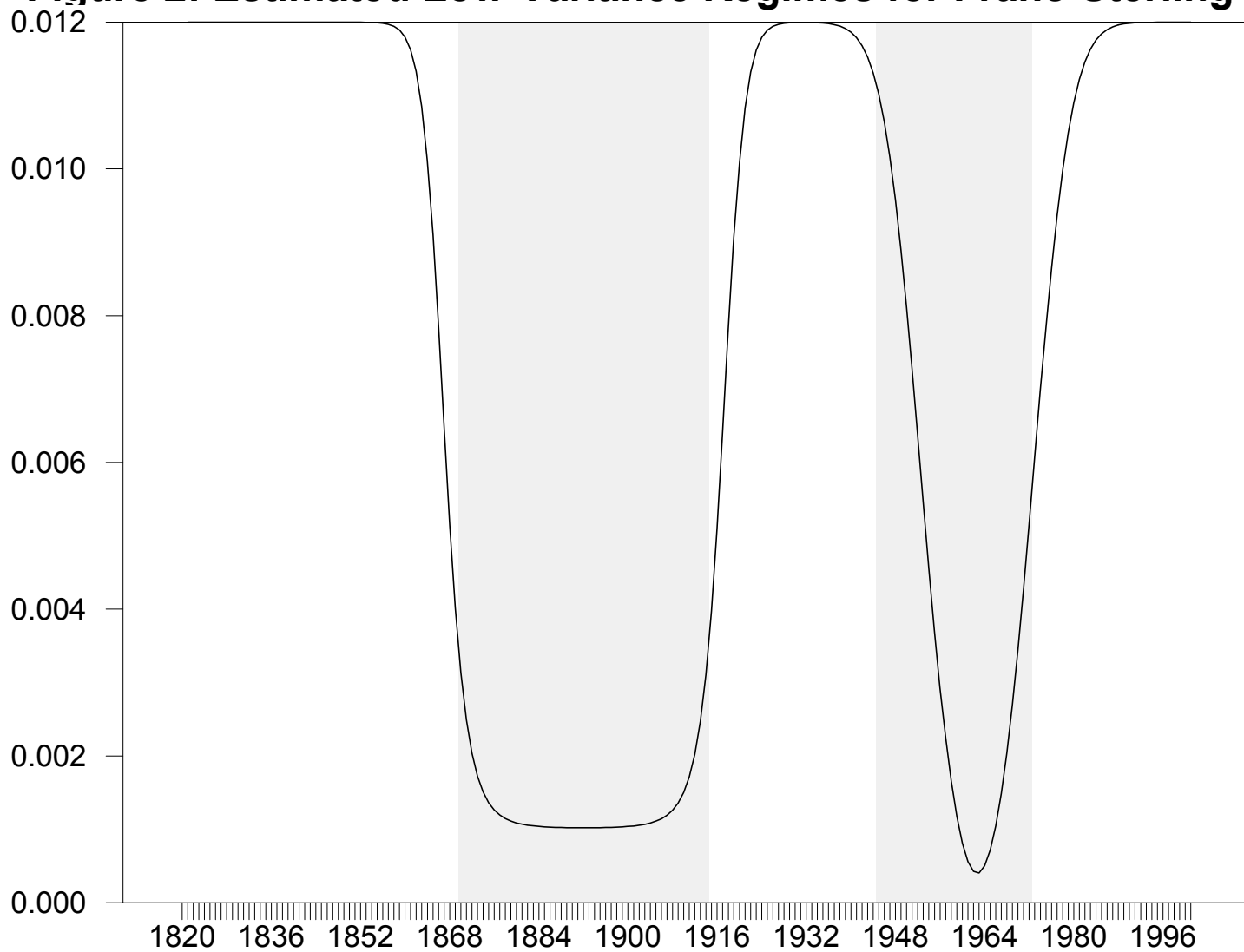


Figure 3: Real Sterling-Dollar and the HBS Effect

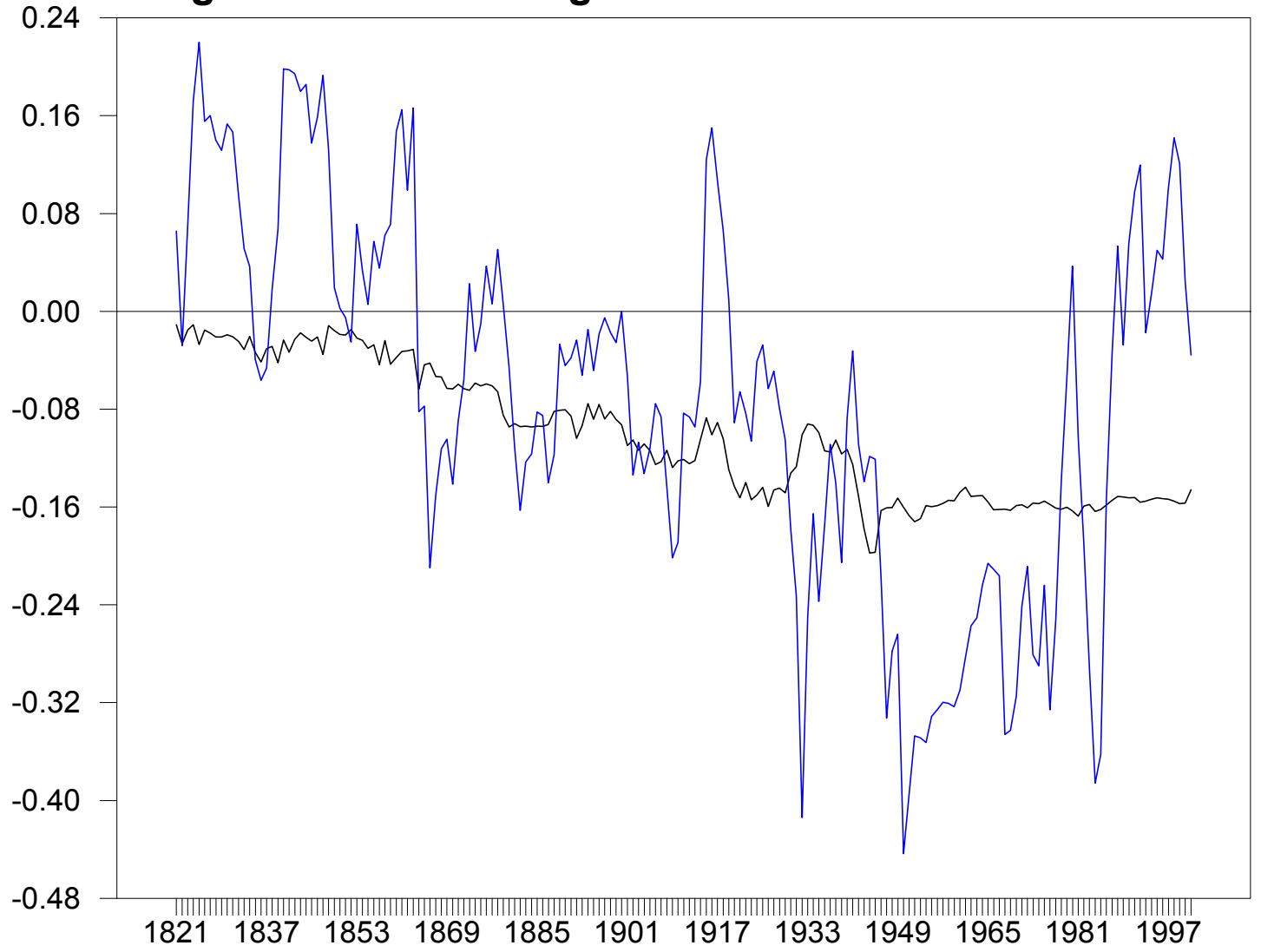


Figure 4: US-UK Prices, Real Exchange Rate and HBS Effect

