

# Real Exchange Rate and Current Account Dynamics with Sticky Prices and Distortionary Taxes

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## Abstract

This paper examines the interaction of real exchange rates and current account movements in open economies subject to monopolistic competition with sticky price-setting behavior and distortionary taxes. We find that the correlations between fiscal balances and the current account depend on the elasticity of net exports with respect to the real exchange rate. Under highly elastic export demand, the welfare effects may be greater or lower than under export demand with a low elasticity.

*Key words:* sticky price setting, current account, real exchange rate

*JEL Classification:* E52, E62, F41

## 1 Introduction

This paper examines the real exchange rate and current account dynamics in an open economy subject to the distortions of monopolistic competition, sticky price setting behavior, and income taxes, with recurring productivity shocks. We find that it matters if exports are sensitive to real exchange rate changes. In particular, the fiscal and current accounts are "twins", or positively correlated, only when export demand is highly elastic with respect to this variable. Otherwise, the fiscal and current account balances are negatively correlated in the presence of continuing productivity shocks. In the latter case, trade deficits simply reflect the response of foreign capital to changes in domestic productivity,

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while fiscal balances increase with the higher tax revenue generated by rising labor income.

The relationship between these two deficits is of more than academic interest. For example, Bradford De Long (2004) notes that "we have a large trade deficit now—and did not back in 1997, because the federal budget deficit is much larger now than it was then." By contrast, former Undersecretary of the Treasury John Taylor (2004) argues that the trade deficit simply reflects the growth of productivity in the United States, leading to capital formation growing faster than U.S. saving. The question comes down to how much fiscal adjustment is in order, when trade deficits start to grow.

Given the monetary and tax regimes in place, the distribution of welfare changes, if the export market becomes more price elastic. This can be good news or bad news, relative to the case of inelastic export demand. But we also argue that the good news is the more likely scenario, since exporting to a market with greater price flexibility may be a backdoor way to import greater price flexibility and lower monopolistic distortions in the domestic market.

In our setup the monetary authority simply targets inflation. This is consistent with recent work on monetary and fiscal interaction in open economies. Kollmann (2004), for example, argues for monetary rules which just respond to inflation and for a tax rate on household income that responds to public debt. He finds that this monetary/fiscal configuration yields welfare results quite close to more elaborate rules. Schmidt-Grohé and Uribe (2004) find that further emphasis on inflation by the monetary authority, beyond what is required for determinacy makes little difference for welfare, while a muted monetary response to output, with passive fiscal rules are best for welfare. Like us, Schmidt-Grohé and Uribe (2004) fully incorporate the distortionary steady-state effects of monopolistic competition in their analysis of monetary and fiscal rules.

Recent work by Razin (2005) has argued that as economies become more open in trade and capital flows, the optimal monetary policy should put progressively more weight on inflation and less weight (or no weight) on output-gap targets. However, Razin eliminated the steady-state distortion of monopolistic competition by a system of taxes and subsidies, and he did not incorporate distortionary taxes and other forms of fiscal policy in his analysis. Yes we find his insight is on target. Even with distortionary income taxes, the best response of monetary policy is to smooth interest rates and respond aggressively to inflation.

Our finding, that correlations of fiscal and current account balances crucially depend on the sensitivity of export demand with respect to the real exchange rate, is consistent with recent work of Bussière, Fratzscher, and Müller (2005). These authors could not detect any robust empirical link between government deficits and the current account, in time series studies of several European countries. Given that the structure of exports markets are beyond the policy scope of a small or medium size country, and that these markets are in a process of change, it should not be surprising that the link between fiscal and current account deficits change through time as well.

Erceg, Guerrieri and Gust (2004) also note that the empirical literature gives

divergent estimates about the effects of fiscal deficits on the trade deficit. Like Bussière, Fratzscher, and Müller, they realized that this issue will not be settled by econometric regression results. Like us, they make use of a stochastic dynamic general equilibrium model, embedding sticky prices as well as other rigidities, to investigate the fiscal/current account linkages. They find, not surprisingly, that the trade price elasticity makes the trade balances more responsive to changes in fiscal balances, but they find that the elasticity has to be implausibly high in order for it to generate a higher response than .2% for a given one percent change in the fiscal deficit. Their model is more complex than the one we use in this book, since it contains many more distortions and rigidities than we have used.

The next section describes the model as well as the monetary/fiscal policy regimes, with calibration based on Smets and Wouters (2002) open-economy version of the Euro-Area model. Then we evaluate the performance of the model with impulse response function for alternative export demand regimes, one with relatively low and one with relatively high elasticity with respect to the real exchange rate. We then conduct accuracy tests and welfare comparisons of regimes with high and relatively low export demand.

## 2 An Open-Economy Model with Sticky Prices

This section presents a simple model of a small open economy. It contains households which are assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models; firms with Calvo-style price-setting behavior and a monetary authority which sets the interest rate using a simply linear Taylor rule.

### 2.1 Households - Consumption and Labor

A representative household, at period 0, optimizes the intertemporal welfare function:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t) \quad (1)$$

$$U_t(.) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varpi}}{1+\varpi} \quad (2)$$

where  $\beta$  is the discount factor,  $C_t$  is an index of consumption goods,  $L_t$  is labour services,  $\sigma$  is the coefficient of relative risk aversion and  $\varpi$  is the elasticity of marginal disutility with respect to labour supply.

The household is assumed to consume only domestically produced goods and to aggregate the bundle of differentiated goods  $j$  using a Dixit-Stiglitz aggregator:

$$C_t = \left[ \int_0^1 (C_{j,t})^{\frac{d-1}{d}} dj \right]^{\frac{d}{d-1}} \quad (3)$$

where  $j$  denotes the domestic goods and the elasticity of substitution is given by  $d > 1$ . Standard cost-minimization yields demand functions:

$$C_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-d} C_t \quad (4)$$

where  $P_t^j$  is the price of each differentiated good and  $P_t$ , the aggregate price level is given by

$$P_t = \left[ \int_0^1 (P_{j,t})^{1-d} dj \right]^{\frac{1}{1-d}}$$

## 2.2 Firms - Production and Pricing

We follow Smets and Wouters (2002) in assuming that each firm  $j$  produces differentiated goods using a Leontief technology:

$$Y_{j,t} = \min \left\{ \frac{v_t L_{j,t}}{(1 - \alpha_y)}, \frac{K_{j,t}}{\alpha_y} \right\} \quad (5)$$

where  $v_t$  is the aggregate productivity shock, which follows the following autoregressive process (in log terms):

$$\begin{aligned} \log(v_t) &= \rho \cdot \log(v_{t-1}) + \epsilon_t \\ \epsilon_t &\sim N(0, \sigma_\epsilon^2) \end{aligned}$$

The symbol  $L^j$  denotes the labor services hired by the firm and  $K^j$  represents the imported intermediate good which is a fixed proportion  $\alpha_y$  of output. Aggregating over all firms yields aggregate supply as:

$$\begin{aligned} Y_t &= \min \left\{ \frac{v_t L_t}{1 - \alpha_y}, \frac{K_t}{\alpha_y} \right\} \\ Y_t &= \left[ \int_0^1 (Y_{j,t})^{\frac{d-1}{d}} dj \right]^{\frac{d}{d-1}} \\ L_t &= \int_0^1 L_{j,t} dj \\ K_t &= \int_0^1 K_{j,t} dj \end{aligned}$$

where  $Y$  is the aggregate domestic output comprising the composite bundle of differentiated goods produced by monopolistically competitive producers. The demand for good  $Y_{j,t}$  is given by the following expression:

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-d} Y_t \quad (6)$$

### 2.3 Price dispersion index and Resource Cost

Both Schmidt-Grohe and Uribe (2004) and Yun (2004) note that sticky price models with staggered pricing, creates a wedge between aggregate supply  $Y$  and aggregate demand. To see this, note first that the demand for good  $i$  is the sum of domestic and foreign demand:

$$Y_{j,t} = C_{j,t} + X_{j,t} \quad (7)$$

Aggregating this over the monopolistic domestic goods producers gives the following relationship between overall output, price dispersion, and the components of aggregate demand,  $C_t$  and  $X_t$  (exports are assumed to be determined exogenously):

$$\begin{aligned} Y_t &= \Delta_t(C_t + X_t + G_t) \\ \Delta_t &= \int \left( \frac{P_{j,t}}{P_t} \right)^{-d} dj \\ \Delta_t &\geq 1 \end{aligned} \quad (8)$$

where where  $\Delta_t \geq 1$  is a measure of relative price dispersion; with  $P_{j,t}/P$  the relative price of firm  $j$  at time  $t$ .

Overall, the major implication of price stickiness is that it creates distortion and hence it generates real resource allocation costs leading to an overall reduction in production (and hence demand for labour services). Briefly, the real resource cost of relative price dispersion - the greater the dispersion of price in the economy, the lower the level of consumption for a given level of aggregate output and export demand. Alternatively, to maintain consumption at a particular level (for a given export demand), the greater the dispersion the greater the demand for labor and intermediate goods:

$$\begin{aligned} L_t &= \frac{(1 - \alpha_y)Y_t}{\Delta_t \nu_t} \\ K_t &= \alpha_y \frac{Y_t}{\Delta_t} \end{aligned}$$

which in turn implies increases in disutility (reduction in welfare) and increases in the current account (and foreign debt).

### 2.4 Embedding Sticky Prices

The key modification made to our model is to drop the assumption that the aggregate price level is always equal to marginal cost:  $P_t = MC_t = (1 - \alpha_y) \frac{W_t}{\nu_t} + \alpha_y P_t^F$ , where, as in Chapter 2,  $P_t$ ,  $P_t^F$ ,  $W_t$ , and  $\nu_t$  represent the domestic price level, the foreign price level in domestic currency, the wage rate, and the productivity index. The coefficient  $\alpha_y$  is from the production function, relating intermediate goods and labor to output.

### 2.4.1 Calvo Price Setting and Markup Distortion

We adopt a version of the Calvo (1983) staggered price system which is summarized in the equations below:

$$P_{j,t}^1 = \left( \frac{P_{j,t-1}}{P_{j,t-2}} \right)^\varkappa P_{j,t-1}, \quad 0 \leq \varkappa \leq 1 \quad (9)$$

$$P_{j,t}^2 = \Psi \frac{Y_{j,t} MC_{j,t} + \sum_{j=1}^{\infty} \left( \frac{1}{\prod_{k=0}^{j-1} (1+R_{t+k})} \right) \xi^j Y_{j,t+j}^i MC_{j,t+j}}{Y_{j,t} + \sum_{j=1}^{\infty} \left( \frac{1}{\prod_{k=0}^{j-1} (1+R_{t+k})} \right) \xi^j Y_{j,t+j}^i} \quad (10)$$

where  $MC_{j,t} = (1 - \alpha_y) \frac{W_t}{v_t} + \alpha_y P_t^F$

$$\Psi = \frac{d}{d-1} \quad (11)$$

Equation (9) describes the backward pricing behavior of firms which did not receive a price-signal. For simplicity, we set the indexation parameter  $\varkappa = 0$ , that is, firms simply keep the price level at the previous period's level. Equation (10) is based on Calvo (1982) and comes from Smets and Wouters (2002). It describes the forward pricing behavior of the remaining firms. This framework was applied by Yun (1996) to business cycles. It represents a first-order condition from maximizing expected profits - a profit function, in which a supplier will change its price at time  $t$  to maximize expected profits, based on the expected duration of the price as well as on expected demand and costs [see Woodford (2003): p. 173-203, for an extensive discussion of this framework]. The term  $MC_t$  represents marginal cost which is identical across firms,  $P_t^F$  is the price of the imported intermediate goods  $P_t^F = P_t^{F*} S_t$  where  $P_t^{F*}$  describes the price set by foreigners which is fully "passed-through" to domestic prices of imported goods. We assume an identical wage  $W_t$ , productivity factor  $v_t$ , foreign price  $P_t^F$ , and production technology,  $\alpha_y$  across all firms,  $MC_{j,t} = MC_t$ . The optimal markup factor,  $\Psi$ , equal to  $\frac{d}{d-1}$ , is derived from maximizing the following profit function of firm  $j$ ,  $\Pi_{j,t}$ , with respect to the price  $P_{j,t}$ :

$$\Pi_{j,t} = P_{j,t} \left( \frac{P_{j,t}}{P_t} \right)^{-d} Y_t - \left( \frac{P_{j,t}}{P_t} \right)^{-d} Y_t \left[ 1 - \alpha_y \right] \frac{W_t}{v_t} + \alpha_y P_t^F \quad (12)$$

Canzoneri, Cumby and Diba (2004) note, marginal revenue divided by price,  $\mathbf{d}[P_{j,t} Y_{j,t}] / P_{j,t}$ , is equal to  $[(d-1)/d] \mathbf{d}Y_{j,t}$ , less than  $\mathbf{d}Y_{j,t}$ , with  $\mathbf{d}$  representing the total differential operator for revenue  $[P_{j,t} Y_{j,t}]$  and output  $Y_{j,t}$ . The factor  $[d/(d-1)]$  is called the markup distortion created by monopolistic competition, and leads firms to produce too little.

We assume that the domestic price level for each of the differentiated goods,  $P_{j,t}$  is a weighted average of a backward-looking price,  $P_{j,t}^1$  with imperfect indexation, and a forward-looking component and  $P_{j,t}^2$  with respective weights of  $\xi$  and  $(1-\xi)$ , with  $\xi$  representing the fraction of goods prices which are

expected to remain unchanged; alternatively that a fraction  $(1 - \xi)$  of firms are forward-looking. For simplicity, the likelihood that any price will be changed in a given period is  $(1 - \xi)$  and it is independent of the length of time since the price was set and the level of the current price. As Woodford (2003, p. 177) notes, while these assumptions are unrealistic, they drastically simplify equilibrium inflation dynamics as well as reduce the state-space required to solve for the dynamics. The aggregate price index is given by the following Dixit-Stiglitz aggregator:

$$P_t = \left[ \xi (P_{t-1})^{1-d} + (1 - \xi) (P_t^2)^{1-d} \right]^{\frac{1}{1-d}} \quad (13)$$

Note that the lagged aggregate price  $P_{t-1}$  in equation 13 replaces  $P_{j,t-1}$ , which appears in equation 9

Equation (13) may also be expressed in the following way:

$$1 = \xi [1 + \pi_t]^{d-1} + (1 - \xi) [p_t^*]^{1-d}$$

where  $p_t^*$  is the relative price ( $P_{j,t}^2/P_t$ ), and  $\pi_t = ((P_t - P_{t-1})/P_{t-1})$  is the aggregate inflation between periods  $t-1$  and  $t$ . Yun (2004) rewrites the dispersion index, in terms of Calvo relative prices, as the following law of motion:

$$\Delta_t = (1 - \xi) [p_t^*]^{-d} + \xi [1 + \pi_t]^d \cdot \Delta_{t-1} \quad (14)$$

Yun (2004) also rewrites the dispersion index, in terms of Calvo relative prices, as the following law of motion:

$$\Delta_t = (1 - \xi) [p_t^{j*}]^{-d} + \xi [1 + \pi_t]^d \cdot \Delta_{t-1} \quad (15)$$

where  $p_t^{j*}$  is the relative price ( $P_t^{j2}/P_t$ ), and  $\pi_t = ((P_t - P_{t-1})/P_{t-1})$  is the aggregate inflation between periods  $t-1$  and  $t$ .

Goodfriend and King (1997) point out that monetary policy cannot eliminate distortion caused by  $\Psi$ , since it is a steady state effect. Studies of optimal monetary policy, evaluating monetary policy rules which compare the dynamics of the model under sticky prices with the dynamics and welfare effects under flexible prices, follow the common practice of eliminating this steady-state distortion by assuming an optimal tax/subsidy scheme to offset the markup effect on pricing and production, in other words,  $\Psi = 1$ . However in this paper, following Schmitt-Grohé and Uribe (2004), we do not eliminate this distortion.

## 2.5 Closure Conditions and Foreign Debt

As Schmidt-Grohé and Uribe (2003) note, without any further modification, the random walk property of this type of models implies an infinite unconditional variance for variables such as  $F$  and  $C$ . To induce stationarity in these variables, several options are available: endogenous discounting, adjustment costs for the accumulation of foreign debt, or the specification of debt-elastic risk premia. Schmidt-Grohé and Uribe find that all of the options deliver "virtually identical" results at business-cycle frequencies.

In this paper we induce stationarity by introducing an asset-elastic interest rate, that is we augment the interest on international asset  $R_t^*$  with a risk premium term  $\Phi_t$  which has the following symmetric functional form:

$$\Phi_t = \begin{cases} -\varphi[\exp(|F_t| - \bar{F})] & \text{if } F_t > \bar{F} \\ \varphi[\exp(|F_t| - \bar{F})] & \text{if } F_t < \bar{F} \end{cases} \quad (16)$$

where  $\bar{F}$  represents the steady-state value of the international asset. If the asset is less (greater) than the steady state, we assume that foreign lenders exact an international risk premium (discount). Note when  $F_t = \bar{F}$  then  $\Phi(F_t) = \varphi[e^{F_t - \bar{F}} - 1] = 0$ . As Schmidt-Grohé and Uribe (2003) note, the value of the coefficient  $\varphi$  directly affects the volatility of the current account to GDP ratio, as well as consumption volatility.

Introducing a risk premium term which is a function of debt  $\Phi(F_t) = \varphi[e^{F_t - \bar{F}} - 1]$  alters the typical Euler equations. In particular, the intertemporal budget equation becomes:

$$\frac{-S_t F_t}{(1 + R_t^* - \Phi(F_t))} + \frac{B_t}{(1 + R_t)} = -S_t F_{t-1} + B_{t-1} + W_t L_t - P_t C_t - Tax_t \quad (17)$$

where  $F$  is a one-period foreign bonds,  $B$  is one-period domestic bonds,  $S$  is the nominal exchange rate (defined as the home currency per unit of foreign),  $W$  is the wage rate,  $P$  is the overall price index,  $R^*$  is the foreign interest rate,  $R$  the domestic interest rate.

## 2.6 Tax Regime and Domestic Debt

We assume that government expenditures equal are pre-set, with  $G = \bar{G}$ . Taxes are levied and collected on real labor income at each period  $t$ :

$$Tax_t = \tau_0 + \tau_L \cdot W_t L_t / P_t \quad (18)$$

where  $\tau_0$  is a lump-sum tax while  $\tau_L$  is the respective tax rate on labor income.

Similarly government debt evolves according to the following equation:

$$\bar{G} - Tax_t = B_t / P_t - B_{t-1} (1 + R_t) / P_t \quad (19)$$

## 2.7 Export Demand and Foreign Debt

The following logarithmic function describes the evolution of exports:

$$\ln(X_t) = \ln(\bar{X}) + \phi_{X,REX} [\ln(S_t / P_t) - \ln(\bar{S} / \bar{P})] \quad (20)$$

where  $\bar{X}$ ,  $\bar{S}$ , and  $\bar{P}$  are the steady state values of exports, the nominal exchange rate, and the price level, and  $\phi_{X,REX}$  is the elasticity of aggregate exports

(relative to steady state levels) with respect to the real exchange rate,  $S_t/P_t$ , relative to its steady state level. Exports thus depend on the current value of the real exchange rate,  $S_t/P_t$ . We could, of course, incorporate J-curve dynamics by putting in lags for the real exchange-rate effect on exports.

Note that we allow for a direct effect of the real exchange rate on exports, we do not allow for such a channel at the import side. There is thus an asymmetry in the treatment of exports and imports.

Given the value of exports ( $X_t$ ) and the imports of intermediate goods ( $K_t$ ) the change in foreign debt evolves as follows:

$$(P_t X_t - P_t^F K_t) = -S_t [F_t - F_t(1 + R_t^* + \Phi(F_{t-1}))] \quad (21)$$

## 2.8 Euler Equations

Maximizing utility subject to the budget constraint, with respect to  $C_t, L_t, B_t$ , and  $F_t$  yields the aggregate first-order Euler equations, given the tax rates  $\tau_{L,t}$  and  $\tau_{C,t}$ :

$$\frac{C_t^{-\sigma}}{P_t} = \lambda_t \quad (22)$$

$$\beta \mathbf{E}_t \lambda_{t+1} = \frac{\lambda_t}{(1 + R_t)} \quad (23)$$

$$\frac{\lambda_t S_t}{(1 + R_t^* - \Phi(F_t))} \left[ \frac{(1 + R_t^* + \Phi(F_t)) - F_t \Phi'(F_t)}{(1 + R_t^* - \Phi(F_t))} \right] = \beta \mathbf{E}_t (\lambda_{t+1} S_{t+1}) \quad (24)$$

$$[1 - \tau_L] \lambda_t W_t = L_t^{\varpi} \quad (25)$$

where  $\mathbf{E}_t$  is the expectations operator conditional on information available at time  $t$ . Note that the tax parameters affect  $\lambda_t$  as well as the consumption Euler equation, given by equation (23) and the labor/real-wage relation, given by equation (25). Of course they also affect equation (24) through  $\lambda_t$  and  $\lambda_{t+1}$ .

Note that the exchange rate is not described by the usual log-linear interest parity formula. If we set  $\Phi(F_t) = 0$ , and assumed  $\mathbf{E}_t (\lambda_{t+1} S_{t+1}) = \mathbf{E}_t (\lambda_{t+1}) \mathbf{E}_t (S_{t+1})$ , log-linearization would produce the familiar interest parity formula, with  $\ln(S_t) = \mathbf{E}_t [\ln(S_{t+1})] + \ln[1 + R_t^*] - \ln[1 + R_t]$ .

## 2.9 Monetary Policy

We assume that the central bank follows a very simple Taylor (1993) rule aimed solely at inflation stabilization,

$$\bar{R} = R^* + \phi_\pi (\pi_t - \tilde{\pi}), \quad \phi_\pi > 1 \quad (26)$$

The actual interest rate follows the following partial adjustment mechanism:

$$R_t = \theta R_{t-1} + (1 - \theta) \bar{R} \quad (27)$$

## 2.10 Evaluation of Export Regimes

We continue with our welfare and utility function:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t) \quad (28)$$

$$U_t(\cdot) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varpi}}{1+\varpi} \quad (29)$$

One well-known way to evaluate alternative price elastic or price-inelastic regimes is to compare the welfare of the sticky-price and tax distorted economy to the welfare of a reference regime  $r$ . The loss function of regime  $i$  can be written in the following way:

$$\ell_t^i = \frac{V_0^i - V_0^r}{V_0^r} \quad (30)$$

where  $V_0^r$  represents welfare in the reference regime  $r$ , and  $V_0^i$  the welfare in policy regime  $i$ . This loss function, of course, is measured in terms of a utility function. Following Schmitt-Grohé, Stephanie and Uribe (2004), the differences in the two welfare indices may be re-expressed as the percentage of consumption that the household in regime  $i$  should be compensated, in order to make the household indifferent between the policy regimes  $i$  and  $r$ . With our utility function, we calculate this consumption compensation percentage in the following way:

$$\ell_0^{C,i} = 100 \left[ 1 - \left( \frac{V_0^i - V_0^r}{\tilde{C}^r} + 1 \right)^{\frac{1}{1-\sigma_C}} \right] \quad (31)$$

$$\tilde{C}^r = \frac{1}{1-\sigma} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t (C_t^r)^{1-\sigma} \quad (32)$$

## 2.11 Parameters

The calibrated values are the same in the previous chapter:

$\sigma = 1.5$	$\beta = 0.99$	$\varpi = 0.25$	$\alpha_y = 0.15$
$\xi = .85$	$d = 6$	$\varphi = .001$	

The values for  $\sigma, \beta, \varpi$  and  $\alpha_y$  are the values suggested by Smets and Wouters (2002). For the Taylor rule parameters, the values for  $\theta, \phi_y$ , and  $\phi_\pi$  will be chosen on a grid to locate the best welfare outcome. The target rate of inflation, in the case of fully flexible prices, is simply zero. Hence  $\tilde{\pi} = 0$ . The Calvo pricing parameters imply a gross mark-up rate of 1.2.

## 2.12 Steady-State Initial Values

Using the normalization, ( $v = 1, \bar{S} = 1.0$ ), the pre-set foreign variables ( $P^{F*} = 1.0, \bar{R}^* = 0.04$ ) and the exogenous variables, ( $\bar{X} = .176, \bar{G} = .15$ ), we solve for the initial steady state values of the other variables ( $C, Y, K, L, W, P, R$ ) and the implied tax rates ( $\bar{\tau}_L$ ) that initial value of foreign and domestic debt are zero ( $\bar{F} = \bar{B} = 0$ ) and the Euler equations are satisfied, as follows:

$$\begin{aligned} (1 - \bar{\tau}_L)W/P &= \frac{((1 - \alpha_y)Y)^\varpi}{(Y - X - G)^{-\sigma}/P} \\ X &= S(\alpha_y Y)/P \\ G &= \bar{\tau}_L W(1 - \alpha_y)Y/P + \bar{\tau}_C(Y - X - G) \end{aligned}$$

We obtained the following values for these initial conditions and parameters:

Steady State Values
Income Tax System
$\bar{Y} = 1.0666$
$\bar{C} = .7333$
$\bar{K} = .1600$
$\bar{X} = .1333$
$\bar{L} = .9066$
$\bar{\tau}_L = .2647$
$U_{ss} = -1.62$

We can, of course, normalize on other initial conditions, with  $C = L = 1$ , with fixed values for  $G$  and  $X$ , across regimes, so that the real wages and the real exchange rates are different, but the utilities and steady-state welfare measures are the same. In this case, we allow a compensation across regimes through real exchange rates and real wages to compensate for the welfare differences of the alternative tax regimes.

In the fully stochastic simulations, in which we examine welfare based on consumption and labor. We note too that this model is specified and calibrated for the case where the steady-state inflation rate is assumed to be zero.

## 3 Solution Algorithm and Decision Rules

We choose to solve the above model with a nonlinear global solution algorithm, based on the collocation projection method. We do not linearize the model, nor do we make use of first or second-order Taylor methods in the popular and widely used perturbation methods[see Collard and Julliard (2001a, 2001b) and Schmidt-Grohé and Uribe (2004a)]. These methods make use of the method of Blanchard and Khan (1985) for rational expectations models with forward and backward-looking variables. As such, they are local solutions while we use a global search method.

In this model we have five state variables, productivity index,  $\nu_t$ , foreign debt  $F_t$ , the price dispersion index, and domestic government debt,  $B_t$  and the interest rate. However, some state variables are more important than others. Given the low inflation in our model, the interest rate and the price dispersion index do not change very much. We found that it makes little difference if we omit them as arguments in the decision rule.

We have the choice of specifying the decision rules for the four forward-looking variables,  $C$ ,  $E$ ,  $VN$ , and  $VD$ , either as a Chebyshev polynomial or as a neural network. Using a Chebyshev second-order polynomial expansion, for three state variables, we have 32 parameters ( $= ndcheb^{nstate} \cdot nddecision.rule$ ), where  $ndcheb$ ,  $nstate$ , and  $nddecision.rule$  represent the degree of the Chebyshev polynomial, the number of state variables, and the number of decision rules, respectively. For the neural network, with two neurons for each decision rule, there also 32 parameters ( $= nneuron \cdot nstate \cdot nddecisionrule + nneuron \cdot nddecisionrule$ ), where  $nneuron$  represents the number of neurons for each decision rule. In this case the number of parameters is the same, given the neural network with two neurons and a second-order polynomial expansion with three state variables. However, as the number of state variables increases, the advantage of the neural network specification over the Chebyshev orthogonal polynomial becomes more apparent. In this paper, we use the neural network specification for the functional form of the decision rules. The advantage, as noted by Sirakaya, Turnovsky, and Alemdar (2005), is that such networks, with logsigmoid functions, easily deliver control bounds on endogenous variables.

The network specification implies the following functional forms for the decision rules for  $C$ ,  $E$ ,  $VN$ , and  $VD$  :

$$\begin{aligned}
\widehat{N}_{1,t}^c &= \psi_{11}^c(F_{t-1}^*) + \psi_{12}^c(v_t^*) + \psi_{13}^c(B_t^*) \\
\widehat{N}_{2,t}^c &= \psi_{21}^c(F_{t-1}^*) + \psi_{22}^c(v_t^*) + \psi_{23}^c(B_t^*) \\
\widehat{C}_t &= \psi_1^{cn} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{1,t}^c)} \right) + \psi_2^{cn} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{2,t}^c)} \right) \\
\widehat{N}_{1,t}^s &= \psi_{11}^s(F_{t-1}^*) + \psi_{12}^s(v_t^*) + \psi_{13}^s(B_t^*) \\
\widehat{N}_{2,t}^s &= \psi_{21}^s(F_{t-1}^*) + \psi_{22}^s(v_t^*) + \psi_{23}^s(B_t^*) \\
\widehat{S}_t &= \psi_1^{ns} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{1,t}^s)} \right) + \psi_2^{ns} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{2,t}^s)} \right) \\
\widehat{N}_{1,t}^{vn} &= \psi_{11}^{vn}(F_{t-1}^*) + \psi_{12}^{vn}(v_t^*) + \psi_{13}^{vn}(B_t^*) \\
\widehat{N}_{2,t}^{vn} &= \psi_{21}^{vn}(F_{t-1}^*) + \psi_{22}^{vn}(v_t^*) + \psi_{23}^{vn}(B_t^*) \\
\widehat{VN}_t &= \psi_1^{n,vn} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{1,t}^{vn})} \right) + \psi_2^{n,vn} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{2,t}^{vn})} \right) \\
\widehat{N}_{1,t}^{vd} &= \psi_{11}^{vd}(F_{t-1}^*) + \psi_{12}^{vd}(v_t^*) + \psi_{13}^{vd}(B_t^*) \\
\widehat{N}_{2,t}^{vd} &= \psi_{21}^{vd}(F_{t-1}^*) + \psi_{22}^{vd}(v_t^*) + \psi_{23}^{vd}(B_t^*) \\
\widehat{VD}_t &= \psi_1^{n,vd} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{1,t}^{vd})} \right) + \psi_2^{n,vd} \cdot \left( \frac{1}{1 + \exp(-\widehat{N}_{2,t}^{vd})} \right)
\end{aligned}$$

The projection method we use involves a search over a wide grid for the state variables, in order to find the values of the coefficients in the decision rules. The search involves a minimization of the Euler equation errors based on a weighted value of the residuals.

Given the nonlinear specification, it is difficult to interpret the magnitudes or signs of the coefficients in the neural network system. So we will not present the estimates of the coefficients given by the projection method. We will instead focus on the economic information available from the impulse response and the stochastic simulations.

## 4 Impulse Response Analysis

To make sure that the calibrated model is stable, and makes sense economically, it is useful to do impulse response analysis. In this case, we set the shock to the log of the productivity coefficient,  $v_t$ , at .1, for period 1, and zero thereafter:

$$\begin{aligned}
\log(v_t) &= \rho \cdot \log(v_{t-1}) + \epsilon_t \\
\epsilon_t &= .1, \quad t = 1 \\
\epsilon_t &= 0, \quad t > 1
\end{aligned}$$

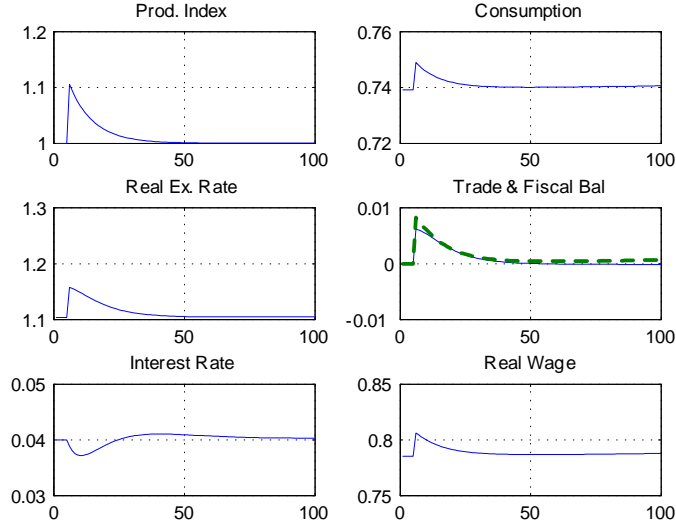


Figure 1: Impulse Response Paths with High Export Elasticity

#### 4.1 Response with High Export Elasticity

Figure 1 pictures the paths of consumption, the real exchange rate, the trade and fiscal balances, the interest rate and the real wages, for a 10 percent productivity shock, under the assumption of relatively high elasticity of exports with respect to the real exchange rate. A temporary increase in the productivity increase leads to temporary increases in consumption, the real exchange rate and real wages, a fall in the interest rate and a rise in the fiscal balance. In our usage, an increase in the real exchange rate is a real depreciation.

We now see that the trade balance also rises. With a relatively strong real exchange rate elasticity, exports rise more than the imports (due to the rising output), so that the current and fiscal accounts are now positively correlated.

#### 4.2 Response with Low Export Elasticity

Figure 2 pictures the same variables under the assumption of a relatively low export elasticity. We see one major difference between 2 and 1. The trade balance now falls after the productivity shock. The rise in imported intermediate goods,  $K$ , is no longer offset by an increase in export demand, so that the productivity increase generates opposite reactions in the fiscal and current-account balances.

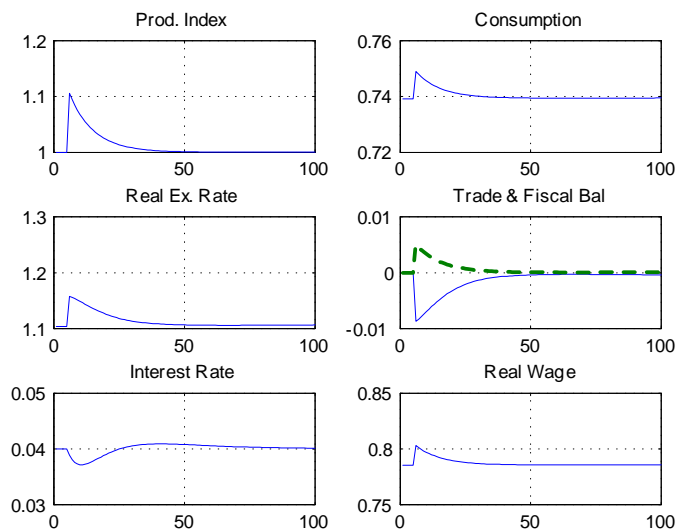


Figure 2: Impulse Response Paths with Low Export Elasticity

### 4.3 Differences

It is hard to tell from 1 and 2 what quantitative differences emerge under the two differing assumptions. Yes, the trade balances move in opposite directions. What about the other variables? Figure 3 shows that real wages are the fiscal balance are slightly higher when exports have a very high real exchange-rate elasticity.

## 5 Stochastic Simulations

This section takes up the accuracy measures of the model, the correlations among key macroeconomic variables, and the welfare consequences of having exports with a relatively high or relatively low price elasticity.

### 5.1 Accuracy Assessment

Before proceeding to our analysis of the correlations of key macroeconomic indicators, we first take up the accuracy of our simulations.

As in previous chapters, we make use of the Judd-Gaspar mean absolute error measures, as well as the Den-Haan and Marcet distributions. Figure 4 pictures the distribution of the Judd-Gaspar error measures for 1000 simulations of sample length 200, under the assumption of a relatively high export price

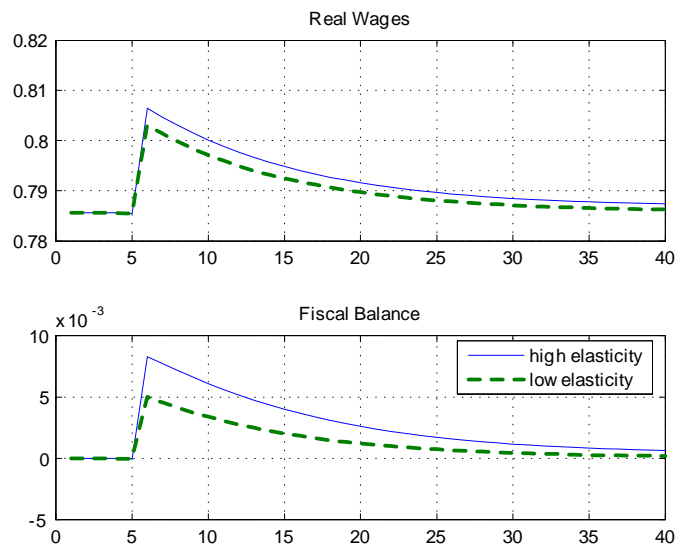


Figure 3: Real Wage and Fiscal Balances Under Alternative Assumptions

elasticity, with  $\phi_{X,REX} = 2.0$ . We see that the mean error measures are less than one cent per dollar of consumption expenditure.

Figure 5 pictures the corresponding error distributions for the case of  $\phi_{X,REX} = .20$ . We see that the distributions are not markedly different.

## 5.2 Correlations

How do the correlations between key macroeconomic variables change with the value of the export price elasticity? Figure 6 pictures the fiscal/trade balance, the real exchange rate/trade balance, the interest rate/real exchange rate, and the interest rate/fiscal balance correlations under the assumption of a relatively high export price elasticity. We see in the lower two quadrants that the correlations are negative: a high interest rate is will likely lead to a real exchange appreciations and a fiscal surplus will lead to lower interest rates. The upper two quadrants show relatively high positive correlations. Given the high export price elasticity, a real exchange rate depreciation leads to a higher trade balance. The fiscal and trade balances are now positively correlated. Given that positive fiscal balances lower interest rates, which in turn lead to a real depreciation, a fiscal surplus goes hand in hand with a trade or current-account surplus.

Figure 7, which gives the corresponding correlations under a relatively low export price elasticity, tells another story. While the correlations in the lower two quadrants remain negative, as above, the correlations between the real exchange rate and trade balance and between the fiscal and trade balances are

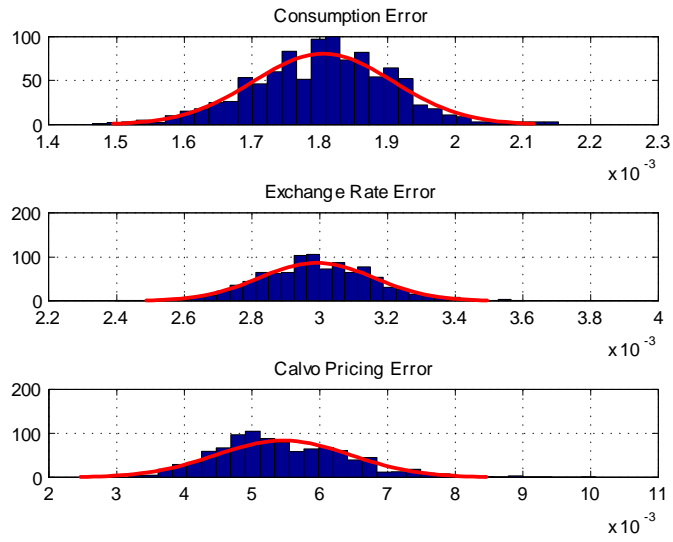


Figure 4: Judd-Gaspar Errors Statistics for High Export Price Elasticity

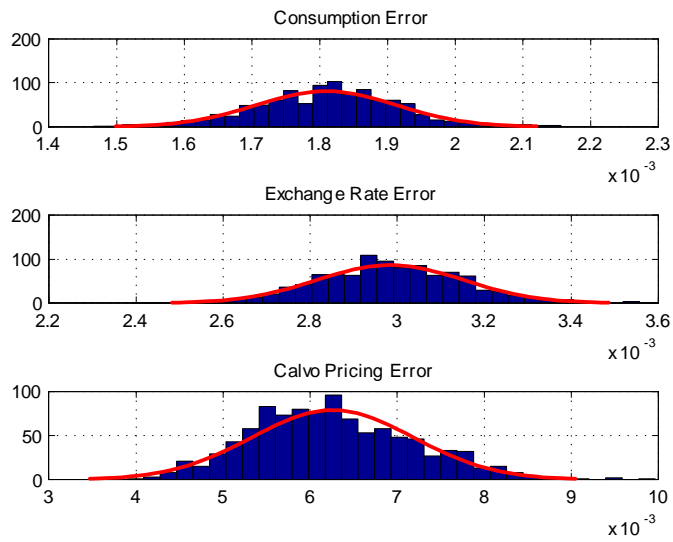


Figure 5: Judd-Gaspar Error Statistics for Low Export Price Elasticity

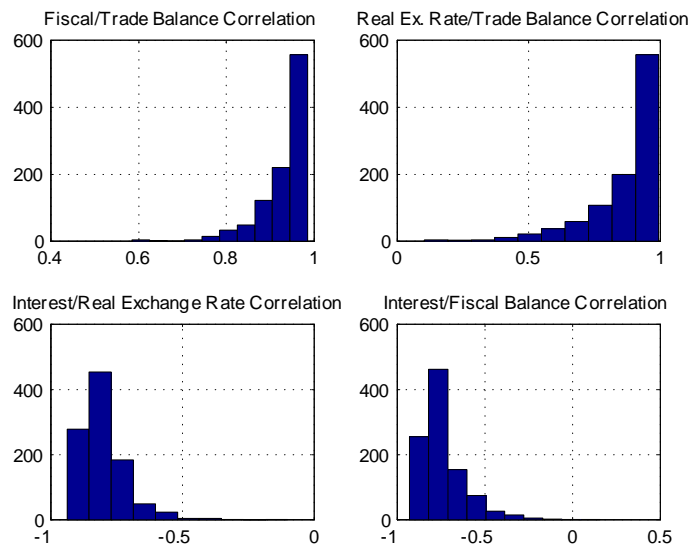


Figure 6: Macroeconomic Correlations Under High Export Price Elasticity

now negative. The key reason is that a real exchange increase, or depreciation, leads to a deterioration in the trade balance. The depreciation in the real exchange rate increases the cost of the imports, since they are used as intermediate goods to produce domestic goods, and exports, while the export demand changes little. Thus, a fiscal surplus, which lowers interest rates and leads to a depreciation, actually worsens the current account.

### 5.3 Welfare Comparisons

When all is said and done, it is better to have exports which have a high or a low price elasticity in foreign markets? In this simple setting, we assume that the export growth or volatility does not feed back into any productivity change for the home country. We assume the same structure of underlying productivity shocks driving the model, whether exports are fixed or variable. This is a drawback, of course, since exporting does generate learning effects which improve domestic productivity.

Figure 8 pictures the welfare distributions under the assumptions of relatively high or relatively low export price elasticity. We see that the variability of the welfare distribution is higher when exports are more price elastic than less price elastic. There is opportunity for welfare gain as well as welfare loss if the exports become more price elastic, due to structural changes in foreign or domestic markets.

Figure 9 pictures the implied consumption compensation between the welfare

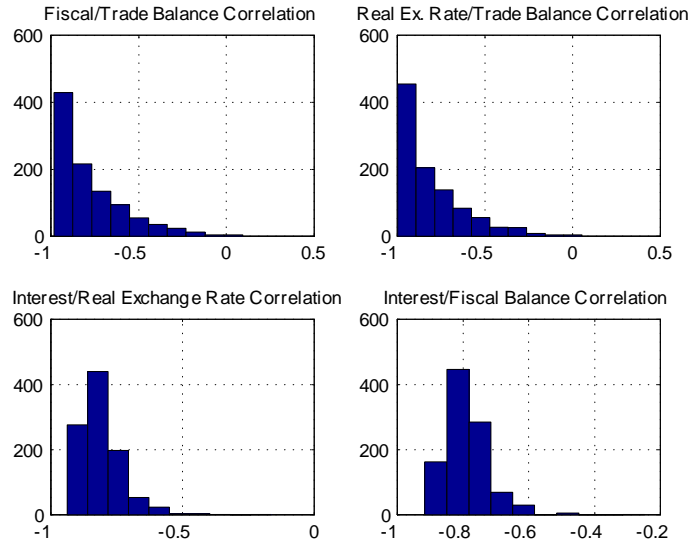


Figure 7: Macroeconomic Correlations Under Low Export Price Elasticity

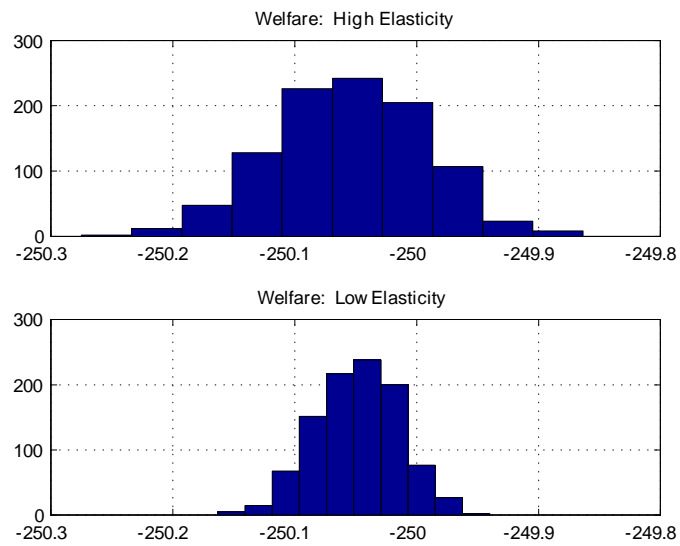


Figure 8: Welfare Distributions Under Alternative Export Price Elasticities

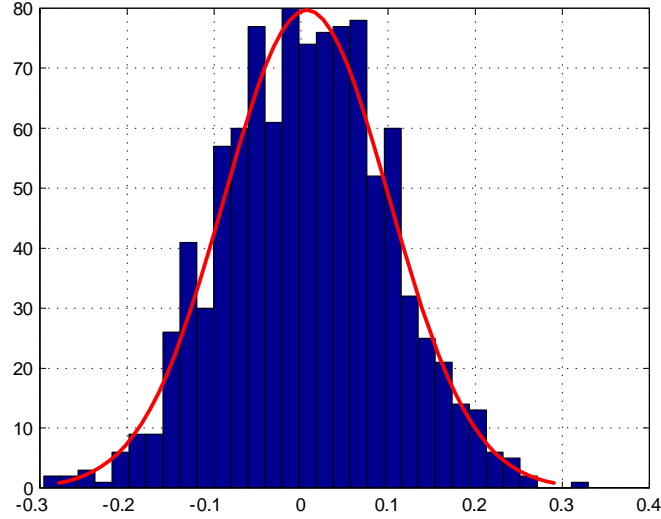


Figure 9: Percentage Consumption Compensation for Welfare Equalization

distributions given in Figure 8. The values are computed with equation (??). We see that the differences amount to at most .3% of a unit of consumption in the reference regime of low export price elasticity. Thus the potential gains or losses are not very large if the structure of the export market changes from a relatively low to a relatively high price elasticity.

## 6 Conclusion

The simulations in this chapter brought up a number of interesting issues, worthy of further exploration. First we see that fiscal and current account deficits may or may not be twins. If there is strong productivity growth and low export-price sensitivity, the current account deficit will likely increase, while the fiscal deficit will shrink. With a high export price elasticity, however, the current account deficit will shrink in tandem with the fiscal deficit as the exchange rate depreciates.

The model incorporates many of the distortions and stickiness popular in the "new neoclassical synthesis" or "new open economy macroeconomics", such as monopolistic competition, sticky price setting behavior, and distortionary taxes. However, we could have added further sources of stickiness, such as imperfect exchange-rate pass through, and sticky wage setting behavior, as well as habit persistence in consumption. We could even allow a given percentage of consumers to be non-Ricardian rule-of-thumb consumers. All of these assumptions

would lower welfare but allow more scope for monetary or fiscal stabilization policy.

We conclude with a recurring theme. As an economy becomes more open, there are opportunities of foreign borrowing or lending, by which consumers may offset the losses of domestic distortions. We see in this paper another benefit of increasing openness or globalization. By exporting to markets where demand is highly price elastic, an economy may be able to import a degree of price flexibility through trade. This price flexibility can, of course, feed back into greater flexibility in domestic markets and thereby further improve welfare. In short, the monopolistic markup factors and the degree of price stickiness may become endogenous.

## References

- [1] Angeloni, Ignazio, Günter Coenen, and Frank Smets (2003), "Persistence, The Transmission Mechanism, and Robust Monetary Policy". *Scottish Journal of Political Economy* 50: 527-549.
- [2] Benigno, Pierpaolo and Michael Woodford (2004), "Optimal Monetary and Fiscal Policy: A Linear Quadratic Approach". Working Paper 345, European Central Bank.
- [3] Blanchard, Olivier and Charles Khan (1985), "The Solution of Linear Difference Equation Models Under Rational Expectations", *Econometrica* 45, 1305-1311.
- [4] Bussière, Matthiew, Marcel Fratzscher, and Gernot J. Müller (2005), "Productivity Shocks, Budget Deficits and the Current Account". Working Paper, European Central Bank.
- [5] Calvo, Guillermo (1983), "Staggered prices in a utility maximising framework", *Journal of Monetary Economics*, 12, 383-398.
- [6] Collard, F. and M. Julliard (2001a), *Perturbation Methods for Rational Expectations Models*. Manuscript: CEPREMAP, Paris.
- [7] \_\_\_\_\_ (2001b), "Accuracy of Stochastic Perturbation Methods in the Case of Asset Pricing Models", *Journal of Economic Dynamics and Control* 25, 979-999.
- [8] Hughes Hallet, Andrew (2005), "Fiscal Policy Coordination with Independent Monetary Policies: Is It Possible?". Working Paper, Department of Economics, Vanderbilt University.
- [9] Judd, John F. and Glenn D. Rudebusch (1998), "Taylor's Rule and the Fed: 1970-1997". *Federal Reserve Bank of San Francisco Economic Review* 3, 3-16.
- [10] Kim, Junill and Sunghyun Henry Kim (2004), "Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation". Working Paper, Department of Economics, Tufts University.

- [11] Kollmann, Robert (2004), "Welfare-Maximizing Operational Monetary and Tax Policy Rules". Working Paper 4782, Center for Economic Policy Research.
- [12] Orphanides, Athanasios and John G. Williams (2002), "Robust Monetary Policy Rules with Unknown Natural Rates". *Brookings Papers on Economic Activity* 2002, 63-118.
- [13] Razin, Asaf (2005), "Globalization and Disinflation: A Note". Working Paper No. 4826, Center for Economic Policy Research.
- [14] Schmidt-Grohé, Stephanie and Martin Uribe (2004a), "Solving Dynamic General Equilibrium Models Using a Second Order Approximation to the Policy Function", *Journal of Economic Dynamics and Control* 28, 755-775.
- [15] \_\_\_\_\_ (2004b), "Optimal Simple and Implementable Monetary and Fiscal Rules" Web page: /www.econ.duke.edu/uribe.
- [16] Smets, Frank and Raf Wouters (2003), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area". European Central Bank, Working Paper 171.
- [17] Taylor, John B. (1993), "Discretion vs. Policy Rules in Practice". *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.
- [18] \_\_\_\_\_ (1999), *Monetary policy rules*, NBER and University of Chicago Press, Chicago.